84. *The Fourier-Borel Transformations of Analytic Functionals on the Complex Sphere*

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1. **Introduction.** Let d be a positive integer and $d\geqslant 2$. $S=S^d$ denotes the unit sphere in $\boldsymbol{R}^{d+1}$. $L(z)$ and $L^*(z)$ denote the Lie norm and the dual Lie norm on $\boldsymbol{C}^{d+1}$ respectively:

$$L(z)=L(x+iy)=[\|x\|^2+\|y\|^2+2\{\|x\|^2\|y\|^2-(x\cdot y)^2\}^{1/2}]^{1/2},$$
$$L^*(z)=\sup\{|\xi\cdot z|;\ L(\xi)\leqslant 1\},$$

where $\xi\cdot z=\sum_{j=1}^{d+1}\xi_j\cdot z_j$, $x, y\in\boldsymbol{R}^{d+1}$, and $\|x\|^2=x\cdot x$.

$\mathcal{O}(\boldsymbol{C}^{d+1})$ denotes the space of entire functions on $\boldsymbol{C}^{d+1}$. We put

$$\operatorname{Exp}(\boldsymbol{C}^{d+1}:(r:N))=\lim_{r'>r}\operatorname{proj} X_{r':N} \qquad \text{for } 0\leqslant r<\infty$$

and

$$\operatorname{Exp}(\boldsymbol{C}^{d+1}:[r:N])=\lim_{r'<r}\operatorname{ind} X_{r':N} \qquad \text{for } 0<r\leqslant\infty,$$

where N is a norm on $\boldsymbol{C}^{d+1}$ and

$$X_{r':N}=\{f\in\mathcal{O}(\boldsymbol{C}^{d+1});\ \sup_{z\in\boldsymbol{C}^{d+1}}|f(z)|e^{-r'N(z)}<\infty\}.$$

We denote the complex sphere by $\tilde{S}=\{z\in\boldsymbol{C}^{d+1};\ z_1^2+z_2^2+\cdots+z_{d+1}^2=1\}$, and we put $\tilde{S}(r)=\{z\in\tilde{S};\ L(z)<r\}$ for $r>1$ and $\tilde{S}[r]=\{z\in\tilde{S};\ L(z)\leqslant r\}$ for $r\geqslant 1$. $\mathcal{O}(\tilde{S}(r))$ denotes the space of holomorphic functions on $\tilde{S}(r)$ and we put $\mathcal{O}(\tilde{S}[r])=\lim\operatorname{ind}_{r'>r}\mathcal{O}(\tilde{S}(r'))$. $\operatorname{Exp}(\tilde{S})$ denotes the restriction to $\tilde{S}$ of the space $\operatorname{Exp}(\boldsymbol{C}^{d+1})$ of entire functions of exponential type. $\mathcal{O}'(\tilde{S}(r))$, $\mathcal{O}'(\tilde{S}[r])$ and $\operatorname{Exp}'(\tilde{S})$ denote the dual spaces of $\mathcal{O}(\tilde{S}(r))$, $\mathcal{O}(\tilde{S}[r])$, and $\operatorname{Exp}(\tilde{S})$ respectively.

The *Fourier-Borel transformation* P_λ for a functional $f'\in\operatorname{Exp}'(\tilde{S})$ is defined by

$$P_\lambda f'(z)=\langle f'_\xi,\ \exp i\lambda(\xi\cdot z)\rangle \qquad \text{for } z\in\boldsymbol{C}^{d+1},$$

where $\lambda\in\boldsymbol{C}$, $\lambda\neq 0$ is a fixed constant.

Morimoto [1] determined the images of $\operatorname{Exp}'(\tilde{S})$ and $\mathcal{O}'(\tilde{S})$ by P_λ. The purpose of this paper is to determine the images of $\mathcal{O}'(\tilde{S}(r))$ and $\mathcal{O}'(\tilde{S}[r])$ by P_λ.

2. **Statement of results.** Our main theorem in this paper is following

Theorem 2.1. *P_λ establishes the following linear topological isomorphisms:*

(2.1) $\quad P_\lambda:\mathcal{O}'(\tilde{S}(r))\xrightarrow{\sim}\operatorname{Exp}_\lambda(\boldsymbol{C}^{d+1}:[|\lambda|r:L^*]) \qquad (r>1),$

(2.2) $\quad P_\lambda:\mathcal{O}'(\tilde{S}[r])\xrightarrow{\sim}\operatorname{Exp}_\lambda(\boldsymbol{C}^{d+1}:(|\lambda|r:L^*)) \qquad (r\geqslant 1),$

where $\operatorname{Exp}_\lambda(\boldsymbol{C}^{d+1}:[|\lambda|r:L^*])=\mathcal{O}_\lambda(\boldsymbol{C}^{d+1})\cap\operatorname{Exp}(\boldsymbol{C}^{d+1}:[|\lambda|r:L^*])$, $\operatorname{Exp}_\lambda(\boldsymbol{C}^{d+1}:(|\lambda|r:L^*))=\mathcal{O}_\lambda(\boldsymbol{C}^{d+1})\cap\operatorname{Exp}(\boldsymbol{C}^{d+1}:(|\lambda|r:L^*))$, *and* $\mathcal{O}_\lambda(\boldsymbol{C}^{d+1})=\{f\in\mathcal{O}(\boldsymbol{C}^{d+1});\ (\Delta_z+\lambda^2)f(z)=0\}$.