82. On Multi-products of Pseudo-differential Operators in Gevrey Classes and its Application to Gevrey Hypoellipticity

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§1. **Introduction and pseudo-differential operators in Gevrey classes.** In [6] we give an estimate of multi-products of pseudo-differential operators with symbols in $S^m_{G(\kappa)}$. This corresponds to the case $\rho=1$ and $\delta=0$ in the sense of Hörmander [3]. In the present paper we treat the general case of (ρ, δ). As an application, we improve a result of Gevrey hypoellipticity obtained by Hashimoto-Matsuzawa-Morimoto [2]. The detailed background and description will be published elsewhere.

The symbols we want to treat in this paper are the following:

Definition. Let $m \in \boldsymbol{R}$, $\kappa \geqq 1$, $\kappa' \geqq 1$, $\theta \geqq 0$ and $0 \leqq \delta \leqq \rho \leqq 1$, $\delta < 1$ with $\kappa(1-\delta) \geqq 1$. We say that a symbol $p(x, \xi)$ belongs to a class $SG^m_{\rho,\delta;\kappa,\kappa',\theta}$ if $p(x, \xi)$ satisfies

i) there exist constants C, M and h such that

$$|p^{(\alpha)}_{(\beta)}(x, \xi)| \leqq CM^{-(|\alpha|+|\beta|)}\alpha!^{\kappa'}(\beta!^{\kappa}+\beta!^{\kappa(1-\delta)}\langle\xi\rangle^{\delta|\beta|})\langle\xi\rangle^{m-\rho|\alpha|} \quad \text{if } \langle\xi\rangle \geqq h|\alpha|^{\theta}, \tag{1}$$

ii) for any multi-index α there exists a constant C_α such that

$$|p^{(\alpha)}_{(\beta)}(x, \xi)| \leqq C_\alpha M^{-|\beta|}(\beta!^{\kappa}+\beta!^{\kappa(1-\delta)}\langle\xi\rangle^{\delta|\beta|})\langle\xi\rangle^{m-\rho|\alpha|} \tag{2}$$

for all x, ξ, where M is a constant independent of α and β.

This definition owes to C. Iwasaki (see also [4]). We also note that the class $SG^m_{\rho,\delta;\kappa,\kappa',\theta}$ contains the class $S^m_{\rho,\delta,\sigma}$ studied in [2] if we set $\kappa = \sigma/(\rho-\delta)$, $\kappa'=1$ and $\theta=\sigma/(\rho-\delta)$ (=their θ).

Let $P=p(X, D_x)$ denote a pseudo-differential operator with a symbol $p(x, \xi) \in SG^m_{\rho,\delta;\kappa,\kappa',\theta}$ defined by

$$Pu=(2\pi)^{-n}\int e^{ix\cdot\xi}p(x, \xi)\hat{u}(\xi)d\xi, \qquad u \in \mathcal{S},$$

where $\hat{u}(\xi)$ is a Fourier transform of u. Then, by the method of [7] we can prove

Proposition 1. *Let $\mathcal{D}^{\{\kappa\}\prime}_{L^2}$ be a class of ultradistributions studied in* [7]. *Then, pseudo-differential operators with symbols in $SG^m_{\rho,\delta;\kappa,\kappa',\theta}$ act on $\mathcal{D}^{\{\kappa\}\prime}_{L^2}$ and their images are also contained in $\mathcal{D}^{\{\kappa\}\prime}_{L^2}$.*

This proposition was first proved by the author in the case of $\rho=1$, $\delta=0$ and by C. Iwasaki in the case of $\delta>0$.

Proposition 2. *Let $\kappa>1$ and let $\mathrm{WF}_{G(\kappa)}(u)$ be the wave front set of u in the Gevrey class of order κ. Assume $\rho>0$ and $\tilde{\kappa} \geqq \max(\kappa, \theta, \kappa'/\rho)$. Then for $p(x, \xi) \in SG^m_{\rho,\delta;\kappa,\kappa',\theta}$ we have*