78. On the Number of Prime Factors of Integers in Short Intervals

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1. Introduction. Let $3 \leqslant n < m$ be integers. Let $\omega(m)$ denote the number of distinct prime factors of m. Let $1 < b(n) \leqslant n$ be a sequence of positive integers. Let $A\{m; \cdots\}$ denote the number of positive integers m which satisfy some conditions. Throughout this paper $p, p_1, p_2, \cdots$ stand for prime numbers and $c_1, c_2, \cdots$ stand for positive constants. We put

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-(1/2)y^2} dy.$$

Then the following result was obtained by Babu [1].

Let $1 \leqslant a(n) \leqslant (\log\log n)^{1/2}$ be a sequence of real numbers tending to infinity. Then

(1) $(1/b(n))A\{m; n < m \leqslant n + b(n),\ \omega(m) - \log\log m < x\sqrt{\log\log m}\} \longrightarrow \Phi(x)$

as $n \to \infty$, provided that $b(n) \geqslant n^{a(n)(\log\log n)^{-1/2}}$.

In this note we shall prove the following theorem which shows that the condition for $b(n)$ can be improved.

Theorem. *Let $\alpha < \beta$ be real numbers. Let $b(n) \geqslant n^{1/(\log\log n)}$ be a sequence of positive integers. We put $\mu = \max\{1, |\alpha|, |\beta|\}$ and*

$$A(n, b(n), \alpha, \beta) = A\{m; n < m \leqslant n + b(n),$$
$$\log\log m + \alpha\sqrt{\log\log m} < \omega(m) < \log\log m + \beta\sqrt{\log\log m}\}.$$

Then we have

$$\frac{1}{b(n)} A(n, b(n), \alpha, \beta) = \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\beta} e^{-(1/2)y^2} dy + O\left(\frac{\mu^5 (\log\log\log n)^{1/2}}{(\log\log n)^{1/4}}\right)$$
$$+ O(\mu\sqrt{\log\log n}\, e^{-c_1 (\log\log n)^2 \log b(n)/\log n}).$$

The O-terms are uniform with respect to a sufficiently large n.

This theorem implies that (1) holds if $b(n) \geqslant n^{1/\log\log n}$, and also gives an answer for the question which was given by P. Erdös and I. Z. Ruzsa (cf. [1]). To prove the theorem we shall use Selberg's sieve method and the arguments of Erdös [3] and Tanaka [5] (cf. [2]).

2. Sieve method. Using Kubilius's lemma (Kubilius [3], lemma 1.4) we obtain the following lemma. This also can be proved directly by Selberg's sieve method.

Lemma. *Let $b_1(n)$ be a sequence of positive integers tending to infinity. Let $g \leqslant \sqrt{b_1(n)}$ be a positive integer and q be an integer with $0 \leqslant q < g$. Let $n_1 = [(n-q)/g]$ and $n_2 = [(n + b_1(n) - q)/g]$, here $[x]$ denotes the largest integer not exceeding x. Let $r_1 \geqslant 2$ with $\log r_1 \leqslant c_2 \log(n_2 - n_1)$, where*