68. *Upper Semicontinuity of Eigenvalues of Selfadjoint Operators Defined on Moving Domains*

By Satoshi KAIZU
Department of Information Mathematics, University of Electro-Communications

(Communicated by Kôsaku YOSIDA, M. J. A., Oct. 14, 1985)

1. Introduction. We are interested in "wild" perturbations in the sense of J. Rauch and M. Taylor [6], on eigenvalue problems for the Laplacian. We show the upper semicontinuity of each k-th eigenvalue of the minus Laplacian with respect to a domain perturbation belonging to a certain class. This class contains a perturbation argued by the author [5]. Hereafter we describe all statements only in an abstract fashion.

Let X and V_ε be real, separable and infinitely dimensional Hilbert spaces with $X \supset V_\varepsilon$. We assume that the injection $V_\varepsilon \to X$ is compact. We denote by $|\ |$ and $(\ ,\ )$ the norm and inner product on X, respectively. Here ε means the value zero or the values of a sequence decreasing to zero. Let $a_\varepsilon : V_\varepsilon \times V_\varepsilon \to \boldsymbol{R}$ be a symmetric continuous bilinear form such that $a_\varepsilon(v) \geqq c_\varepsilon \|v\|_V^2$ for all $v \in V_\varepsilon$, where $a_\varepsilon(v) = a_\varepsilon(v, v)$ and c_ε is a positive constant. We denote by H_ε the closure of V_ε in X and denote by P_ε the orthogonal projection from X onto H_ε. We set $\Sigma = \{x \in X \mid |x| = 1\}$. We define a positive selfadjoint operator $A_\varepsilon : D(A_\varepsilon) \to H_\varepsilon$ by $a_\varepsilon(u, v) = (A_\varepsilon u, v)$ for all $u \in D(A_\varepsilon)$ and $v \in V_\varepsilon$, where $D(A_\varepsilon) = \{u \in V_\varepsilon \mid \exists c > 0$ such that $|a_\varepsilon(u, v)| \leqq c|v|$ for all $v \in V_\varepsilon\}$. We consider the equation: $A_\varepsilon u_\varepsilon = \mu_\varepsilon u_\varepsilon$, $\mu_\varepsilon \in \boldsymbol{R}$ and $u_\varepsilon \in \Sigma$. Let $\mu_\varepsilon^{(k)}$ be the k-th eigenvalue of A_ε counting with its multiplicity; $0 \leqq \mu_\varepsilon^{(1)} \leqq \mu_\varepsilon^{(2)} \leqq \cdots$ and $\mu_\varepsilon^{(k)} \to \infty$ as $k \to \infty$. We have

$$\text{(1)} \qquad \begin{aligned} \mu_\varepsilon^{(1)} &= \inf_{V_\varepsilon \cap \Sigma \ni x} a_\varepsilon(x) \\ \mu_\varepsilon^{(k)} &= \sup_{\substack{H_\varepsilon \ni x_i \\ 1 \leqq i \leqq k-1}} \ \inf_{\substack{V_\varepsilon \cap \Sigma \ni x \\ (x, x_i) = 0, 1 \leqq i \leqq k-1}} a_\varepsilon(x) \quad k \geqq 2 \end{aligned}$$

(cf. R. Courant and D. Hilbert [4]). If $\varepsilon = 0$ then we drop from V_ε, H_ε, A_ε, P_ε and so on. Next we describe our result.

Theorem 1. *If*

$$\text{(2)} \qquad \operatorname*{s-lim}_{\varepsilon \to 0} (1 + \lambda A_\varepsilon)^{-1} P_\varepsilon = (1 + \lambda A)^{-1} P$$

for a certain $\lambda > 0$. *Then we have* $\limsup_{\varepsilon \to 0} \mu_\varepsilon^{(k)} \leqq \mu^{(k)}$ *for each* $k \in N$.

Remark 2. Rauch and Taylor [6] discussed in detail various concrete domain perturbations for the Laplacian, which assure (2), although the domain perturbation of [5] is not treated by [6]; theorem 4.1 of L. Boccardo and P. Marcellini [3] also describes the asymptotic properties of eigenvalues of the Laplacian (cf. theorem 3.71 of H. Attouch [1]), but we can not apply this theorem to the perturbation of [5]. However, the