54. *A New Formulation of Local Boundary Value Problem in the Framework of Hyperfunctions.* III

Propagation of Micro-Analyticity up to the Boundary

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In our previous notes ([4] and [5]) we have formulated boundary value problems in a unified way both for systems of linear partial differential equations with non-characteristic boundary and for single equations with regular singularities. In [5] we have also microlocalized this formulation. As an application of this microlocal formulation we present here some new results on propagation of micro-analyticity of solutions up to the boundary for equations satisfying a kind of micro-hyperbolicity; we can treat a class of equations for which the boundary is totally characteristic as well.

First let us briefly recall some of the definitions and results of [4] and [5]. Put

$$M=\boldsymbol{R}^n \ni x=(x_1, x'), \quad X=\boldsymbol{C}^n \ni z=(z_1, z'), \quad z'=(z_2, \cdots, z_n),$$
$$N=\{x \in M\,;\, x_1=0\}, \quad Y=\{z \in X\,;\, z_1=0\}, \quad \tilde{M}=\boldsymbol{R}\times \boldsymbol{C}^{n-1},$$
$$M_+=\{x \in M\,;\, x_1\geqq 0\}, \quad \tilde{M}_+=\{(x_1, z') \in \tilde{M}\,;\, x_1\geqq 0\}.$$

We set $\mathcal{B}_{N|M_+}=(\iota_*\iota^{-1}\mathcal{B}_M)|_N$, where $\mathcal{B}_M$ is the sheaf of hyperfunctions on M and ι: int $M_+\to M$ is the natural embedding. We use the notation $D=(D_1, D')$, $D'=(D_2, \cdots, D_n)$ with $D_j=\partial/\partial z_j$. We have defined in [5] a sheaf $\mathcal{C}_{M+}$ on $S^*_M\tilde{M}$ and put $\mathcal{C}_{N|M+}=\mathcal{C}_{M+}|_{L_0}$ with $L_0=S^*_M\tilde{M}|_N\cong S^*_NY$. There is an exact sequence

$$0\longrightarrow \tilde{\iota}_*\tilde{\iota}^{-1}\mathcal{B}\mathcal{O}|_N\longrightarrow \mathcal{B}_{N|M+}\longrightarrow (\pi_{N/Y})_*\mathcal{C}_{N|M+}\longrightarrow 0,$$

where $\tilde{\iota}$: int $\tilde{M}_+\longrightarrow \tilde{M}$ is the embedding, $\pi_{N/Y}$: $S^*_NY\to N$ is the projection, $\mathcal{B}\mathcal{O}$ is the sheaf on $\tilde{M}$ of hyperfunctions with holomorphic parameters z'.

Let $\mathcal{M}$ be a coherent $\mathcal{D}_X$-module (i.e. a system of linear partial differential equations with analytic coefficients) defined on a neighborhood in X of $\mathring{x}=(0, \mathring{x}') \in N$. First we assume

(N.C) Y is non-characteristic for $\mathcal{M}$.

Then there exist injective sheaf homomorphisms

$$\gamma:\ \mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{B}_{N|M_+})\longrightarrow \mathcal{H}om_{\mathcal{D}_Y}(\mathcal{M}_Y, \mathcal{B}_N),$$
$$\gamma:\ \mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{C}_{N|M_+})\longrightarrow \mathcal{H}om_{\mathcal{D}_Y}(\mathcal{M}_Y, \mathcal{C}_N)$$

compatible with each other, where $\mathcal{M}_Y$ is the tangential system of $\mathcal{M}$ to Y (Corollary of [4] and Theorem 3 of [5]).

Next let us assume

(R.S) $\mathcal{M}=\mathcal{D}_X/\mathcal{D}_XP$ with $P=a(x)((z_1D_1)^m+A_1(z, D')(z_1D_1)^{m-1}+\cdots+A_m(z, D'))$; here $a(z)$ is a holomorphic function with $a(\mathring{x})\neq 0$,