53. *On Strong Hyperbolicity for First Order Systems*

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§1. **Introduction.** In this note, we shall study strong hyperbolicity for first order hyperbolic systems;

$$L(x, D)=-D_0+\sum_{j=1}^{d} A_j(x)D_j+B(x),$$

where $A_j(x)$, $B(x)$ are $N\times N$ matrices with smooth entries defined near the origin in R^{d+1} with coordinates $x=(x_0, x')=(x_0, x_1, \cdots, x_d)$ and $D_j=-i(\partial/\partial x_j)$. Denote $\xi=(\xi_0, \xi')=(\xi_0, \xi_1, \cdots, \xi_d)$ and by $h(x, \xi)$ the determinant of the principal symbol $L_1(x, \xi)$ of $L(x, D)$;

$$L_1(x, \xi)=-\xi_0+\sum_{j=1}^{d} A_j(x)\xi_j,$$

and say that $L_1(x, \xi)$ is strongly hyperbolic if the Cauchy problem for $L(x, D)$ is C^∞ well posed near the origin for any lower order term $B(x)$ ([8]). Throughout this paper, we assume that $h(x, \xi)$ is hyperbolic with respect to dx_0 near the origin, i.e. $h(x, \xi_0, \xi')=0$ has only real roots for any (x, ξ'), $\xi'\in R^d\backslash 0$, $x\in R^{d+1}$ (x near the origin) and furthermore we assume that the multiplicities of these characteristic roots are at most two.

We shall prove that if $L_1(x, \xi)$ is strongly hyperbolic near the origin then at every point $(x, \xi)\in T^*R^{d+1}\backslash 0$ (x near the origin), $L_1(x, \xi)$ is effectively hyperbolic or diagonalizable (that is similar to a diagonal matrix). Conversely when $L_1(x, \xi)$ is effectively hyperbolic at every $\rho=(\bar{x}, \bar{\xi})$ with $\pi(\rho)=(\bar{x}, \bar{\xi}')$, we know that for any $B(x)$, there is a parametrix of $L(x, D)$ near $(\bar{x}', \bar{\xi}')$ with finite propagation speed of wave front sets ([10]), where π is the projection from T^*R^{d+1} to $R\times T^*R^d$ off ξ_0. In case $L_1(x, \xi)$ is diagonalizable near every ρ with $\pi(\rho)=(\bar{x}, \bar{\xi}')$, we shall show, under some additional conditions, that $L_1(x, \xi)$ is smoothly symmetrizable near $(\bar{x}, \bar{\xi}')$. Hence for any $B(x)$, $L(x, D)$ has a parametrix near $(\bar{x}', \bar{\xi}')$ with finite propagation speed of wave front sets.

§2. **Notations and results.** Let $L_0(x, \xi)$ be the symbol of degree 0 of $L(x, \xi)$, ${}^{co}L_1(x, \xi)$ the cofactor matrix of $L_1(x, \xi)$, and $L^s(x, \xi)$ the subprincipal symbol of $L(x, \xi)$;

$$L^s(x, \xi)=L_0(x, \xi)+\frac{i}{2}\sum_{j=0}^{d}(\partial^2/\partial\xi_j\partial x_j)L_1(x, \xi).$$

We denote by $F(\rho)$ the Fundamental (Hamilton) matrix corresponding to the Hessian Q of $h/2$ at ρ and set

$$Tr^+h(\rho)=\sum \mu_j$$