51. On the Positive Solutions of an Emden-Type Elliptic Equation

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§1. **Problem and main results.** Recently, many authors have been studying the problem of existence and uniqueness of positive entire solutions for second order semilinear elliptic equations. In this note we investigate the existence of positive entire solutions for the boundary value problem,

$$(\mathrm{E})\quad \begin{cases} (1.1) & \Delta u+\phi(|x|)u^m=0, \qquad x\in R^n, \\ (1.2) & u(x)\to b \qquad \text{as } |x|\to\infty, \end{cases}$$

where $n\geq 3$, $\Delta=\sum_{i=1}^n \partial^2/\partial x_i^2$, $|x|=(x_1^2+\cdots+x_n^2)^{1/2}$ and b is a given nonnegative constant. We assume that $\phi(r)$ satisfies

$$(\mathrm{H})\quad \phi\in C(R_+),\quad \phi\geq 0 \text{ on } R_+,\quad \phi\not\equiv 0 \text{ and } \int_0^\infty r\phi(r)dr<\infty.$$

Here and hereafter, R_+ denotes the interval $[0, \infty)$.

It has already been shown by Kawano [1] that under the condition (H), (1.1) has infinitely many positive entire solutions. A similar result has also been obtained by Ni [2] under slightly stronger condition. Here we study the boundary value problem (E). Naito [3] and Fukagai [4] have investigated several subjects related to this problem. Our main results are as follows:

Theorem 1. *Let $m>1$. Suppose* (H) *and*

(H_1) $m<(n+2+2l)/(n-2)$, *where* $\phi(r)=c_l r^l+o(r^l)$ *at* $r=0$ *for some* $l\geq 0$ *and* $c_l>0$,

are satisfied. Then there exists some $B>0$ *such that*

(i) *for any* $b\in(B,\infty)$, (E) *has not any radially symmetric positive solution,*

(ii) *for any* $b\in[0,B]$, (E) *has a radially symmetric positive solution.*

Theorem 2. *Let $m<1$. Suppose* (H) *is satisfied. Then* (E) *has a radially symmetric positive solution for any* $b\in[0,\infty)$.

In the case of $m>1$, (H_1) seems to be crucial for the existence of positive solutions of (E) with $b=0$. The following is an example which breaks the condition (H_1) and always has strictly positive solutions, if we restrict ourselves to radially symmetric solution;

$$(1.3)\qquad \Delta u+\phi(|x|)u^5=0, \qquad x\in R^3,$$

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