38. A Monotone Boundary Condition for a Domain with Many Tiny Spherical Holes

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1. Introduction. Let $\boldsymbol{R}^N$ be divided into an infinitely many number of cubes C_ε^i, $i\in N$, with volume of $(2\varepsilon)^N$. Let $B^i(r_\varepsilon)$ be a closed ball of radius $r_\varepsilon(<\varepsilon)$ set in the center of C_ε^i, here $N\geqq 3$. Let Ω be a bounded domain with smooth boundary Γ. We denote by F_ε the union of all balls $B^i(r_\varepsilon)$ $(\subset\Omega)$ such that dist $(B^i(r_\varepsilon),\Gamma)\geqq\varepsilon$. Let $\Omega_\varepsilon=\Omega\backslash F_\varepsilon$. Let ν be the outer unit normal of $\partial\Omega_\varepsilon$. For a positive number L_ε and a non-negative number c_ε we consider a monotone function β_ε defined by (i) $\beta_\varepsilon(r)=(r+c_\varepsilon)/L$ for $r\leqq -c_\varepsilon$, (ii) $\beta_\varepsilon(r)=0$ for $|r|\leqq c_\varepsilon$, (iii) $\beta_\varepsilon(r)=(r-c_\varepsilon)/L$ for $r\geqq c_\varepsilon$. In this paper we regard functions of $L^2(\Omega_\varepsilon)$ as functions of $L^2(\Omega)$ vanishing outside Ω_ε. For $f\in L^2(\Omega)$ we consider the boundary value problem:

(1) $$-\Delta u_\varepsilon=f \qquad \text{a.e. in } \Omega_\varepsilon$$

(2) $$\frac{\partial u_\varepsilon}{\partial\nu}+\beta_\varepsilon(u_\varepsilon)=0 \qquad \text{a.e. on } \partial\Omega_\varepsilon.$$

The problem admits a unique solution $u_\varepsilon\in H^2(\Omega_\varepsilon)$ (cf. [2]). We consider the behavior of u_ε under the condition

(3) $$\sup L_\varepsilon<\infty,\ c_\varepsilon\to 0,\ r_\varepsilon\to 0 \quad \text{and} \quad n_\varepsilon\to\infty$$

where n_ε is the number of holes of Ω_ε. Let $|\Omega|$ be the measure of Ω. In this paper the relation $n_\varepsilon\sim|\Omega|/(2\varepsilon)^N$ as $\varepsilon\to 0$ is very often used. Let b be a multivalued monotone function defined by (iv) the domain $D(b)=\{0\}$, (v) $b(0)=\boldsymbol{R}$. Replacing (2) by $\partial u_\varepsilon/\partial\nu+b(u_\varepsilon)\ni 0$ we obtain the Dirichlet boundary value problem.

The behavior of the Laplacian on domains with many tiny spherical holes, concerning the Dirichlet boundary condition, has been studied by M. Kac [3], J. Rauch and M. Taylor [6], S. Ozawa [5], D. Cioranescu and F. Murat [1] and other authors. Among them we shall extend the result of Cioranescu and Murat to the direction of the monotone boundary condition (2). Intuitively we have $\beta_\varepsilon\to b$ as $L_\varepsilon\to 0$ and $c_\varepsilon\to 0$. Thus the above idea may be natural. For another extension see S. Kaizu [4].

Theorem. *Let u_ε be the solution of* (1), (2) *and let $\tilde{u}_\varepsilon\in H^1(\Omega)$ be an extension of u_ε to be harmonic in F_ε. Take constants p,q such that $0\leqq p<\infty$ and $0\leqq q\leqq\infty$. We assume that the parameters r_ε, n_ε, c_ε and L_ε vary with* (3) *and*

(4) $$\sup c_\varepsilon/r_\varepsilon<\infty,\ n_\varepsilon r_\varepsilon^{N-2}\to p \quad \text{and} \quad L_\varepsilon/r_\varepsilon\to q.$$

Then $\tilde{u}_\varepsilon$ converges weakly in $H^1(\Omega)$ to the solution of