37. CR-Microfunctions and the Henkin-Ramirez Reproducing Kernel

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§ 0. The method of integral representations is natural and very useful in the theory of functions of several complex variables. One of the most important integral representations was given by Henkin [4] and by Ramirez [9]. They succeeded in constructing a holomorphic reproducing kernel for strictly pseudo-convex domains by using the so-called Cauchy-Fantappiè formula.

On the other hand, the curvilinear wave expansion (Radon transformation) due to Sato-Kawai-Kashiwara, which is a natural generalization of the plane wave decomposition of the δ-function, is a fundamental tool in the theory of linear partial differential equations. For, by using this expression, one can reproduce holomorphic functions which define a given hyper (or micro) function (Kataoka [6]). Bony [2] noticed that this curvilinear wave expansion was constructed from the Cauchy-Fantappiè kernel by taking its boundary values to the real domain as local cohomology.

In this paper, we study the boundary values of the Henkin-Ramirez kernel from the microlocal point of view and verify in particular that the Henkin-Ramirez kernel is a holomorphic reproducing kernel of CR-microfunctions (=microfunction solutions of the tangential Cauchy-Riemann equations). We also show at the same time that the way of construction of the Henkin-Ramirez kernel and that of Sato-Kawai-Kashiwara's curvilinear wave expansion of the δ-function are essentially of the same type from the microlocal point of view.

This simple observation makes it possible in a unified manner to construct holomorphic reproducing kernels for CR-microfunctions on a certain class of CR-submanifolds. This generalization will be dealt in the subsequent paper.

§ 1. Let D be a strictly pseudo-convex domain in C^n with real analytic boundary ∂D; $D=\{z \mid \rho(z)<0\}$, where ρ is a real analytic defining function such that $d\rho(z)$ does not vanish on ∂D. Since we are interested in the (micro-) local properties, we only consider the Levi polynomial L_ρ associated with ρ, which is an analytic family of local holomorphic support functions defined by

$$L_\rho(z, z')=\sum_{j=1}^{n} \frac{\partial \rho}{\partial z'_j}(z')(z'_j-z_j)-\frac{1}{2} \sum_{j,k=1}^{n} \frac{\partial^2 \rho}{\partial z'_j \partial z'_k}(z')(z'_j-z_j)(z'_k-z_k).$$