36. Fourier Transform of a Space of Holomorphic Discrete Series

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(Communicated by Kôsaku YOSIDA, M. J. A., May 13, 1985)

1. Let G be a connected non-compact real simple Lie group of matrices and K a maximal compact subgroup of G. Assume G/K is a hermitian symmetric space. Then, G/K can be realized as a Siegel domain D of type II. Let $\mathfrak{h}$ be a Cartan subalgebra of $\mathfrak{g}=\mathrm{Lie}\, G$ contained in $\mathfrak{k}=\mathrm{Lie}\, K$, Δ the root system of $(\mathfrak{g}_C, \mathfrak{h}_C)$. We introduce an order in Δ compatible with the complex structure of G/K. For each K-dominant K-integral linear form Λ on $\mathfrak{h}_C$ satisfying Harish-Chandra's non-vanishing condition [1, p. 612], the holomorphic discrete series Π_Λ of G is realized on a Hilbert space $\mathcal{H}(\Lambda)$ (see 5) of vector valued holomorphic functions on D. Let $S(D)$ be the Šilov boundary of D. Then, one knows that $S(D)$ is diffeomorphic to a certain nilpotent subgroup $N(D)$ of the affine automorphisms of D. By identifying $S(D)$ with $N(D)$, the aim of this note is a description of the space $\mathcal{H}(\Lambda)$ by using the Fourier transform on $N(D)$. If D reduces to a tube domain, $N(D)$ is abelian. Since such a description in this case is found in [6], we assume from now on that D does not reduce to a tube domain. Then, $N(D)$ is a simply connected 2-step nilpotent Lie group.

2. Let $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ be a Cartan decomposition of $\mathfrak{g}$ and $\mathfrak{p}_+$ (resp. $\mathfrak{p}_-$) the sum of all root subspaces corresponding to positive (resp. negative) non-compact roots in Δ. $\mathfrak{p}_\pm$ are abelian subalgebras of $\mathfrak{g}_C$ normalized by $\mathfrak{k}_C$. Let $P_\pm$ and K_C be analytic subgroups of G_C ($\mathrm{Lie}\, G_C=\mathfrak{g}_C$) corresponding to $\mathfrak{p}_\pm$ and $\mathfrak{k}_C$ respectively. Every $x\in P_+K_CP_-$ can be expressed in a unique way as $x=\exp\zeta_+\cdot k(x)\cdot\exp\zeta_-$ with $\zeta_\pm\in\mathfrak{p}_\pm$, $k(x)\in K_C$. We know that G is contained in $P_+K_CP_-$. Let $\{\gamma_1, \cdots, \gamma_l\}$ be a maximal system of positive non-compact strongly orthogonal roots such that for each j, γ_j is the largest positive non-compact root strongly orthogonal to $\gamma_{j+1}, \cdots, \gamma_l$. For every $\alpha\in\Delta$, we choose $X_\alpha\in\mathfrak{g}_\alpha$ as in Lemma 3.1 in [2, p. 257]. Then,

$$\mathfrak{a}=\sum_{1\leq i\leq l}\mathbf{R}(X_{\gamma_i}+X_{-\gamma_i})$$

is a maximal abelian subspace of $\mathfrak{p}$ with l=real rank of G. Let

$$c=\exp\pi\sum_{1\leq j\leq l}(X_{\gamma_j}-X_{-\gamma_j})/4\in P_+K_CP_- \tag{1}$$

and $\nu=\mathrm{Ad}\, c$. As we are assuming that G/K does not reduce to a tube domain, there is only one possibility of positive $\mathfrak{a}$-root system $\Phi(\mathfrak{a})^+$ compatible with the original order in Δ through ν^* [3, p. 364]: put $2\lambda_j=\nu^*(\gamma_j)$, then

$$\Phi(\mathfrak{a})^+=\{\lambda_i+\lambda_j\,;\,1\leqq j\leqq i\leqq l\}\cup\{\lambda_i-\lambda_j\,;\,1\leqq j<i\leqq l\}\cup\{\lambda_i\,;\,1\leqq i\leqq l\}.$$