34. On the Cauchy Problem for Effectively Hyperbolic Systems

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1. Introduction. In this note, we study the Cauchy problem for the first order hyperbolic systems which are effectively hyperbolic, that is the determinant of the principal part of these systems is effectively hyperbolic. Using the same method in [4], [6], we shall show that the Cauchy problem for such systems is C^∞ well posed for any choice of lower order terms, and give a theorem of propagation of wave front sets which is analogous to the result in [5], [7] for effectively hyperbolic operators.

In what follows, we use the following notation,

$$x=(x_0, x'), \quad x'=(x_1, \cdots, x_d), \quad \xi=(\xi_0, \xi'), \quad \xi'=(\xi_1, \cdots, \xi_d),$$
$$D_j=-i(\partial/\partial x_j), \qquad D'=(D_1, \cdots, D_d).$$

We study the Cauchy problem for $L(x, D)$ with the symbol

$$L(x, \xi)=\sum_{j=0}^{m} A_j(x, \xi')\xi_0^{m-j}, \qquad A_0(x, \xi')=I_N, \tag{1.1}$$

where $A_j(x, \xi')$ are $(N\times N)$ classical pseudo-differential symbols of degree j defined in a conic neighborhood W of $(0, \bar{\xi}')$ in the cotangent bundle $T^*\boldsymbol{R}^d$, depending smoothly on x_0 in an open interval I containing the origin and I_N is the identity matrix of degree N.

Denote by $h(x, \xi)$ the determinant of the principal part $L_0(x, \xi)$ of $L(x, \xi)$. We shall say that $L_0(x, \xi)$ is effectively hyperbolic system at $\rho=(0, \bar{\xi}) \in \boldsymbol{R}\times T^*\boldsymbol{R}^d$ if $h(x, \xi)$ satisfies the following conditions,

(1.2) $h(x, \xi)$ is hyperbolic with respect to dx_0, that is the equation $h(x, \xi_0, \xi')=0$ has only real roots in ξ_0 for any (x, ξ') near ρ,

(1.2) if $dh(0, \xi_0, \bar{\xi}')=0$, then the fundamental matrix $F_h(0, \xi_0, \bar{\xi}')$ has non zero real eigenvalues,

(for the definition of the fundamental matrix, see [1], [2]). Then we have

Theorem 1.1. *Assume that $m=1$ and $L_0(x, \xi)$ is effectively hyperbolic at $\rho=(0, \bar{\xi}')$. Then, in a sufficiently small conic neighborhood of $(0, \bar{\xi}') \in T^*\boldsymbol{R}^d$, there is a parametrix of $L(x, D)$ with finite propagation speed of wave front sets.*

From this Theorem, it follows that

Theorem 1.2. *Assume that $m=1$ and $L_0(x, \xi)$ is effectively hyperbolic system at every $(0, \xi')$ $(|\xi'|=1)$. Then the Cauchy problem for $L(x, D)$ is locally solvable in the C^∞ class in a neighborhood of the origin in $\boldsymbol{R}^{d+1}$ with the data on $x_0=0$.*

Remark 1.1. Parametrices in conic open sets with finite propagation speed of WF are defined in [4] for scalar operators. A generalization of