11. *Small Deformations of Certain Compact Class L Manifolds*

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The notion of Class L manifolds was introduced by Ma. Kato [1]. The most significant property of Class L is that any two members of Class L can be connected complex analytically to obtain another Class L manifold. The purpose of this note is to construct a series of compact Class L 3-folds $\{M(n)\}_{n\in N}$ inductively and to determine their all small deformations. Details will be published elsewhere.

1. We denote the 3-dimensional complex projective space by $\boldsymbol{P}^3$ of which the system of homogeneous coordinates we write $[\zeta_0:\zeta_1:\zeta_2:\zeta_3]$. For any positive real number r, we define a domain U_r in $\boldsymbol{P}^3$ by $U_r=\{[\zeta_0:\zeta_1:\zeta_3:\zeta_3]\in \boldsymbol{P}^3 \,|\, |\zeta_0|^2+|\zeta_1|^2<r(|\zeta_2|^2+|\zeta_3|^2)\}$. A complex 3-fold X is said to be of *Class L* if it contains a domain which is biholomorphic to U_1, in other words, if there exists a holomorphic open embedding of U_1 into X. Let σ be a holomorphic automorphism of $\boldsymbol{P}^3$ defined by $\sigma([\zeta_0:\zeta_1:\zeta_2:\zeta_3])=[\zeta_2:\zeta_3:\zeta_0:\zeta_1]$. For any real number ε greater than 1, we denote the domain $U_\varepsilon-\overline{U_{1/\varepsilon}}$ by $N(\varepsilon)$ where $\overline{}$ indicates the topological closure. Then it is easy to see that U_r is isomorphic to $U=U_1$ and that $\sigma(N(\varepsilon))=N(\varepsilon)$.

Suppose that X_1 and X_2 are Class L manifolds with open embeddings $i_\nu: U_\varepsilon\to X_\nu$, $\nu=1,2$. Put $X_\nu^\#=X_\nu-\overline{i_\nu(U_{1/\varepsilon})}$. We define a complex manifold $Z(X_1,X_2,i_1,i_2)=X_1^\#\cup X_2^\#$ by identifying a point $x_1\in i_1(N(\varepsilon))\subset X_1^\#$ with the point $x_2=i_2\circ\sigma\circ i_1^{-1}(x_1)\in X_2^\#$. $Z(X_1,X_2,i_1,i_2)$ is also a Class L manifold because $N(\varepsilon)$ is of Class L. Remark that the construction of $Z(X_1,X_2,i_1,i_2)$ depends on the choice of the open embeddings i_1 and i_2.

Now we define a compact Class L manifold $M=M(1)$. Let l_0 and l_∞ be projective lines in $\boldsymbol{P}^3$ given by

$$l_0:\zeta_0=\zeta_1=0,\qquad l_\infty:\zeta_2=\zeta_3=0,$$

and put $W=\boldsymbol{P}^3-l_0-l_\infty$. Consider the holomorphic automorphism g: $[\zeta_0:\zeta_1:\zeta_2:\zeta_3]\mapsto[\zeta_0:\zeta_1:\alpha\zeta_2:\alpha\zeta_3]$ of W, where α is a complex number with $0<|\alpha|<1$. Letting $\langle g\rangle$ be the infinite cyclic group generated by g, we define the complex manifold M to be the quotient space of W by $\langle g\rangle$. Taking real numbers β,γ,δ such that $|\alpha|<\beta<\gamma<\delta<1$, we define subdomains U_0, U_W, U_∞ in W as follows: