81. *Parametrices and Propagation of Singularities near Gliding Points for Mixed Problems for Symmetric Hyperbolic Systems. I*

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1. Introduction. Let $P(x, D)$ be a symmetric hyperbolic system defined on R^{n+1} in the form:

$$P(x, D)=\sum_{k=0}^{n} A_k(x)D_k+C(x), \qquad D_k=-i\partial/\partial x_k,$$

where $x=(x_0, x_1, \cdots, x_n)$, $A_k(x)$ are hermitian $m\times m$ matrices and $A_0(x)$ is positive definite. Consider the following mixed problem in the closed half space $X=\{x=(x', x_n)\,;\, x_n\geqq 0,\ x'=(x_0, x_1, \cdots, x_{n-1})\in X'=R^n\}$ with boundary ∂X:

(1) $\quad P(x, D)u=0$ in X, $B(x)u=f$ on ∂X, $u(x)=0$ in $X\cap\{x_0\ll 0\}$.

In a previous paper [5] we proved, under certain restrictions on P and B, that there exists a parametrix near the diffractive point. The purpose of this note is to show the existence of a parametrix near the gliding point and study the propagation of singularities. Such results have been obtained by Eskin [2] and Petkov [8] in the case where the uniform Lopatinski condition (see (3) below) is satisfied, and by Taylor and Melrose [10] in a case, analogous to the Neumann problem for d'Alembertian where this condition is violated, which is a special case of ours. When $f=0$ but the initial data do not always vanish, the propagation of singularities has been investigated by several authors ([1], [3], [4] and [7]).

It seems that, to show the existence of a parametrix near a gliding point, one needs to make use of an Airy function $A_0(z)=Ai(-z)$ having zeros on the positive real axis. This causes difficulties which do not appear in the diffractive case. In fact, the boundary condition leads to an equation on the boundary which involves a Fourier integral operator with singular phase function. Moreover, when one solves the equation in a (not conic) region near the glancing surface, a pseudodifferential operator belonging to a bad class $OPS_{0,0}$ appears. Furthermore, if P is not strictly hyperbolic, in addition, if there exist two or more waves associated with a gliding ray, one can not reduce the equation to another which involves only one unknown.

As is seen in [2] or [9], the uniform Lopatinski condition guarantees that one can overcome these difficulties. Now suppose this is violated. In order to derive a basic a priori estimate for the equation