75. *A Family of Finite Nilpotent Groups*

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1. Introduction. The primary purpose of this paper is to show that Theorems 1 and 2 in our previous work [3] can be extended to a much wider class of p-genera of capitulation than of regular ones, as was mentioned there in Remark 3. But we shall be concerned here thoroughly with finite nilpotent groups.

As far as transfers of a p-group G to its normal subgroups are concerned, it was confirmed in [2] that we have

$$V_{G\to N}(g)=g^{[G:N]}\cdot[N, N]$$

for every $g\in G$ and every normal subgroup N of G if G is regular. Here $V_{G\to N}$ is the transfer of G to N and $[N, N]$ denotes the commutator subgroup of N. In this paper, we show that this phenomenon on transfers appears in the nilpotent groups of a wider family than that of regular groups. In fact, this new family is closed under the operation of taking direct products though the direct product of two regular p-groups is not necessarily regular in general (e.g. Weichsel [4]). It is also closed under the operation of taking quotient groups. But it should be noted that it is not closed under taking (normal) subgroups. We shall give a method of constructing members of the new family from a special type of p-groups which do not belong to the family, and see that there are a lot of irregular p-groups in the family even if $p=2$.

2. The property TNP of finite nilpotent groups. Let G be a finite nilpotent group.

Definition. G has the property TNP, or is a TNP-group if the *transfer* of G to every *normal* subgroup N of G coincides with the $[G:N]$-th *power* map modulo $[N, N]$, or in other words, if we have

$$V_{G\to N}(g)=g^{[G:N]}\cdot[N, N]\qquad \text{for } \forall g\in G$$

for every normal subgroup N of G.

Proposition 1. *A quotient group of a TNP-group is a TNP-group.*

Proof. Let G be a TNP-group, and M be a normal subgroup of G. Put $\bar{G}=G/M$. Then every normal subgroup $\bar{N}$ of $\bar{G}$ corresponds to a normal subgroup N of G containing M. Then $N\backslash G$ and $\bar{N}\backslash\bar{G}$ are canonically isomorphic. Therefore, by the definition of transfers, we have the commutative diagram,