71. *Teichmüller Spaces of Seifert Fibered Manifolds with Infinite* π_1

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(Communicated by Shokichi IYANAGA, M. J. A., Sept. 12, 1984)

It is known that geometric structure which 3-manifolds can possess is one of $\mathrm{H}^3, \mathrm{E}^3, \mathrm{S}^3, \mathrm{H}^2\times \boldsymbol{R}, \mathrm{S}^2\times \boldsymbol{R}, \widetilde{\mathrm{SL}}_2$, Nil, Sol, ([9]). Teichmüller space of a geometric manifold M is the set of all metric (of the geometry) on M factored by isotopy. The topology is the quotient of C^∞-topology. For H^3, if M is a Haken 3-manifold, the Teichmüller space is trivial by Mostow's rigidity theorem. In this note we determine Teichmüller spaces of geometric 3-manifolds modelled on $\mathrm{H}^2\times \boldsymbol{R}, \widetilde{\mathrm{SL}}_2, \mathrm{E}^3$, Nil, $\mathrm{S}^2\times \boldsymbol{R}$. We denote the Teichmüller space of M by $\mathcal{T}(M)$. Throughout this note M is compact and orientable.

§1. **Teichmüller spaces of 2-orbifolds.** As geometric manifolds modelled on $\mathrm{H}^2\times \boldsymbol{R}, \widetilde{\mathrm{SL}}_2, \mathrm{E}^3$, Nil, $\mathrm{S}^2\times \boldsymbol{R}$ are Seifert fibered manifolds, we consider Teichmüller spaces of base orbifolds first.

Theorem 1. *Let O be a compact hyperbolic 2-orbifold (possibly nonorientable with geodesic boundaries) with k cone points and without other singularities. Then $\mathcal{T}(O)\cong \boldsymbol{R}^{-3\chi(X)+2k}$ where X denotes the underlying space of O.*

The theorem above appears in Thurston [8] with the sketch of the proof in the case that O is closed orientable.

Theorem 2. *The Teichmüller spaces of Euclidean 2-orbifolds are as follows:*

O (2-*orbifold*)	$\mathcal{T}(O)$
Torus, S^2 *with* 4 *cone points*	$\boldsymbol{R}^3$
Annulus, Möbius band, Klein bottle	$\boldsymbol{R}^2$
D^2 *with* 2 *cone points*, P^2 *with* 2 *cone points*	$\boldsymbol{R}^2$
S^2 *with* 3 *cone points*	$\boldsymbol{R}$

§2. **Teichmüller spaces of geometric manifolds modelled on** $H^2\times \boldsymbol{R}, \widetilde{\mathrm{SL}}_2, \mathrm{E}^3$, **Nil.**

Lemma 1 (Waldhausen [10]). *Let M be a Haken Seifert fibered manifold which is neither of $S^1\times S^1\times I$, $S^1\times S^1\times S^1$, the twisted I-bundle over Klein bottle, the double of the twisted I-bundle over Klein bottle, solid torus. Then the fibration of M is unique up to isotopy.*

Lemma 2 (P. Scott [7]). *Let M be a Seifert fibered manifold whose base orbifold is $S^2(p,q,r)$ where $p,q\geqq 4$. Let $f: M\to M$ be a homeomorphism homotopic to the identity. Then f is isotopic to the identity.*