66. *Continuity of the Inverse of a Certain Integral Operator*

By Yoshio HAYASHI
College of Science and Technology, Nihon University

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§ **1.** Let $L=\bigcup L_j$ be a union of a finite number of simple, smooth and bounded open arcs in $\mathbf{R}^2$, where any two of L_j have neither an interior point nor an end point in common. Denote points in $\mathbf{R}^2$ by x, y, etc., and the distance between x and y by $|x-y|$. Let $\partial L=\{x^*\}$ be the set of end points x^* of L, and set $\bar{L}=L\cup\partial L$. Suppose $C=C(\bar{L})$, $C^\infty=C^\infty(\bar{L})=\mathcal{E}(\bar{L})$, $C_0^\infty=C_0^\infty(\bar{L})=\mathcal{D}(\bar{L})$, etc., represent the function spaces on $\bar{L}$ in the usual sense.

Assume $\psi(x, y)=(1/4i)H_0^{(2)}(k|x-y|)$, where $H_0^{(2)}$ is the zero-th order Hankel function of the second kind, and k is a constant such as $\operatorname{Im} k \leqq 0$. ψ is a fundamental solution of the Helmholtz equation.

We shall define an integral operator Ψ by

$$\Psi\tau\equiv\int_L \psi(x, y)\tau(y)ds_y \tag{1}$$

and denote the inverse of Ψ by Ψ^{-1}. The purpose of this work is to study about the continuity of Ψ^{-1}.

Since $\psi(x, y)$ has only a log singularity at $x=y$, Ψ maps $C(\bar{L})$ into $C(\bar{L})$. Furthermore, as was proved in the previous paper [1], $\Psi\tau=0$ is equivalent to $\tau=0$. However, as is implied by the Riemann-Lebesgue theorem, Ψ^{-1} is not necessarily continuous. For example, for $x\neq a$, we have

$$\int_0^a \psi(x, y)\cos my\, dy=\left(\frac{1}{m}\right)\psi(x, a)\sin ma-\left(\frac{1}{m}\right)\int_0^a \frac{\partial\psi(x, y)}{\partial y}\sin my\, dy.$$

The right hand side exists in the sense of Cauchy's principal value of integral, which tends to zero as $m\to\infty$. However, $\cos mx$ does not tend to zero in $C([0, a])$. In contrast with this, we shall show that Ψ^{-1} is continuous if Ψ is considered to map $\mathcal{D}\to\mathcal{E}$.

§ **2. Definition 1.** Set $\psi(x, y)=\psi_0(x, y)=\psi^{[0]}(x, y)$, where ψ is the one defined above, and set

$$\psi_m(x, y)=\int^{s_y} \psi_{m-1}(x, z)ds_z,$$

and

$$\psi^{[m]}(x, y)=\frac{\partial}{\partial s_x}\int^{s_y} \psi^{[m-1]}(x, z)ds_z, \qquad (m=1, 2, \cdots),$$

where $\int^{s_y}\{\quad\}ds_z$ is the integration with respect to the arc element ds_z