64. *On Capitulation of Ideals of an Algebraic Number Field*

By Katsuya MIYAKE
Department of Mathematics, College of General Education, University of Nagoya

(Communicated by Shokichi IYANAGA, M. J. A., June 12, 1984)

1. Introduction and the main result. Let k be a fixed algebraic number field of finite degree, and K be an unramified abelian extension of k. We denote the absolute ideal class groups of k and K by $C\ell(k)$ and by $C\ell(K)$, respectively. Let $\lambda_{K/k}$: $C\ell(k) \to C\ell(K)$ be the homomorphism defined naturally by lifting ideals of k to the ones of K, and put $P_k(K) = \mathrm{Ker}(\lambda_{K/k})$. Then this is the subgroup of $C\ell(k)$ consisting of those classes the ideals of which become principal in K. Let $S_k(K)$ be the unramified abelian extension of k corresponding to $P_k(K)$ by class field theory. Among the family of unramified abelian extensions of k, the members of the form $S_k(K)$ are very special. Our concern in this note is to characterize these members.

Since K and $S_k(K)$ are abelian over k, they are expressed as the compositions of the maximal p-subextensions $K^{(p)}$ and $S_k^{(p)}(K)$, respectively, for a prime p running over the prime divisors of $|C\ell(k)|$. Since we can show $S_k^{(p)}(K) = S_k^{(p)}(K^{(p)})$ (Proposition 1), we may restrict ourselves to p-extensions of k for a fixed prime p.

Let $\mathcal{K}^{(p)} = \mathcal{K}^{(p)}(k)$ be the family of all unramified abelian p-extensions of k. For $K \in \mathcal{K}^{(p)}$, the maximal unramified abelian p-extension of K is denoted by $\tilde{K}$. Then $\tilde{K}$ is the genus field of the relative abelian extension $\tilde{k}/K$ in the sense of Furuta [2]. Put $\tilde{\mathcal{K}}^{(p)} = \tilde{\mathcal{K}}^{(p)}(k) = \{\tilde{K} \mid K \in \mathcal{K}^{(p)}\}$. For our purpose, it is natural to classify the members of $\mathcal{K}^{(p)}$ using $\tilde{\mathcal{K}}^{(p)}$. The subfamily of $\mathcal{K}^{(p)}$ determined by $L \in \tilde{\mathcal{K}}^{(p)}$ as $\mathcal{G}_L^{(p)} = \mathcal{G}_L^{(p)}(k) = \{K \in \mathcal{K}^{(p)} \mid \tilde{K} = L\}$ will be called a p-genus of capitulation over k, or simply, a p-cap.-genus. A p-cap. -genus $\mathcal{G}_L^{(p)}$ will be called regular if the p-group $\mathrm{Gal}(L/k)$ is regular. (See Hall [3, § 4] or Huppert [4, Ch. III, § 10].)

The main result of this note is

Theorem 1. *Suppose that the p-cap.-genus $\mathcal{G}_L^{(p)}(k)$ with $L \in \tilde{\mathcal{K}}^{(p)}(k)$ is regular. Then for $K \in \mathcal{G}_L^{(p)}(k)$, $S_k^{(p)}(K)$ is determined by L and the degree $[K:k]$; more precisely, we have*

$$\mathrm{Gal}(L/S_k^{(p)}(K)) = \{\sigma \in \mathrm{Gal}(L/k) \mid \sigma^{[K:k]} = 1\}.$$

An immediate consequence of the theorem is

Theorem 2. *Let K_1 and K_2 be unramified abelian p-extensions of*