61. *Invariants of Reductive Lie Groups of Rank One and Their Applications*

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§1. **Introduction.** Throughout this note, G will denote a reductive complex linear algebraic group. A representation of G is a finite dimensional vector space V over the complex number field $\mathbf{C}$ together with a morphism of algebraic groups $\varphi: G \to GL(V)$. We will consider φ together with the representation space V and denote a representation as above by φ, by (φ, G) or by $\varphi(G)$. Let $\mathbf{C}[\varphi]$ be the coordinate ring of the affine space φ on which G acts naturally and let $\mathbf{C}[\varphi]^G$ be the $\mathbf{C}$-subalgebra consisting of all invariant polynomials in $\mathbf{C}[\varphi]$ under this action of G. (φ, G) is said to be *completely co-intersected* (abbrev. COCI) if $\mathbf{C}[\varphi]^G$ (and so φ/G) is a complete intersection. Recall that (φ, G) is said to be *coregular* if $\mathbf{C}[\varphi]^G$ is a polynomial ring over $\mathbf{C}$. All coregular representations of simple algebraic groups were determined in [2, 10].

When Gx is closed in φ for an element x in φ, the isotropy group G_x is reductive, and we call the natural representation of G_x on $T_xV/T_x(G_x)$ *the slice representation at* x, which is denoted by φ_x. Then $\varphi_x/G_x \to \varphi/G$ is etale at the image of x in φ/G ([6]), and we easily get

Lemma (1.1). *Every slice representation of a COCI representation of G is COCI.*

As any representation of G is completely reducible, [11, (5.2)] implies

Lemma (1.2). *Every subrepresentation of a COCI representation of G is COCI.*

These lemmas are useful in studying COCI representations of reductive groups.

§2. **Reductive groups of rank one.** In this section, we suppose that $\operatorname{rank} G=1$. Let T be a maximal torus of G and $\nu: \operatorname{Hom}(T, \mathbf{C}^*) \xrightarrow{\sim} \mathbf{Z}$ a fixed isomorphism. For a representation ρ of G, let ρ^+ (resp. ρ^-) be the direct sum of all ρ_χ with $\nu(\chi)>0$ (resp. $\nu(\chi)<0$), where ρ_χ is the subspace of ρ of weight $\chi \in \operatorname{Hom}(T, \mathbf{C}^*)$. Moreover, put $q_T(\rho)=\min\{\dim \rho^+, \dim \rho^-\}$ and $p_T(\rho)=\min\{|\{\chi \mid \rho_\chi \neq 0, \nu(\chi)>0\}|, |\{\chi \mid \rho_\chi \neq 0, \nu(\chi)<0\}|\}$.

Theorem (2.1). *Let ρ be a representation of G and suppose that (ρ, G) is COCI. Then:*