5. Global Existence Theorem for Nonlinear Wave Equation in Exterior Domain

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The global existence of solutions for the nonlinear wave equation has been extensively studied. For the Cauchy problem Klainerman [1] has made a remarkable improvement recently. That is, he showed that if the spatial dimension is not smaller than 6 and initial data are small and smooth, then the Cauchy problem for the fully nonlinear wave equation has a unique classical global solution. On the other hand it is important to consider the initial boundary value problem for the nonlinear wave equation in an exterior domain in order to study scattering of a reflecting object for the nonlinear wave equation. In the present paper we shall announce that if the spatial dimension is not smaller than 3 and initial data are small and smooth, then we have the global unique existence theorem of classical solutions for a large class of nonlinear wave equations in exterior domains with the homogeneous Dirichlet boundary condition.

Let Ω be an unbounded domain in $\boldsymbol{R}^n$, $n \geqq 3$, with its boundary $\partial\Omega$ C^∞ and compact. We denote a time variable by t or x_0 and a space variable by $x=(x_1, \cdots, x_n)$, respectively. We abbreviate $\partial/\partial t$, $\partial/\partial x_j$ and $(\partial/\partial x_1)^{\alpha_1} \cdots (\partial/\partial x_n)^{\alpha_n}$ to ∂_t or ∂_0, ∂_j and ∂_x^α, respectively, where α is a multi-index with $|\alpha|=\alpha_1+\cdots+\alpha_n$ and $j=1, \cdots, n$. We consider the following problem:

$$\text{(M.P)} \qquad \begin{aligned} &\Phi(u)=\Box u+F(t, x, \Lambda u)=f(t, x) \qquad \text{in } [0, \infty)\times\Omega, \\ &u=0 \qquad \text{on } [0, \infty)\times\partial\Omega, \\ &u(0, x)=\phi_0(x), \quad (\partial_t u)(0, x)=\phi_1(x) \qquad \text{in } \Omega, \end{aligned}$$

where $\partial_t^2-\Delta=\partial_t^2-\sum_{j=1}^n \partial_j^2$ and $\Lambda u=(\partial_i u, i=0, \cdots, n; \partial_j\partial_k u, j, k=0, \cdots, n)$.

Before we state assumptions and the main theorem, we list notations. For p with $1 \leqq p \leqq \infty$ we denote the standard L^p space defined on Ω and its norm by $L^p(\Omega)$ and $\|\cdot\|_p$, respectively. For a vector valued function $h=(h_1, \cdots, h_s)$ we put

$$\|h\|_p=\sum_{j=1}^{s} \|h_j\|_p.$$

For a positive integer N we put

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