90. A Shape of Eigenfunction of the Laplacian under Singular Variation of Domains

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Recently the author has studied a sharp asymptotic behaviour of eigenvalues of the Laplacian under singular variation of domains. See Ozawa [3]–[6]. See Matsuzawa-Tanno [1], Mazja-Nazarov-Plamenevskii [2], for other related topics. In this note we will give a new formula for eigenfunctions of the Laplacian concerning singular variation of domains.

Let Ω be a bounded domain in $\boldsymbol{R}^3$ with smooth boundary $\partial\Omega=\gamma$. Let w be a fixed point in Ω. Let B_ε be the ball defined by $B_\varepsilon=\{z\in\Omega\,;\,|z-w|<\varepsilon\}$ and let $\Omega_\varepsilon=\Omega\backslash\bar{B}_\varepsilon$. Then, the boundary of Ω_ε consists of γ and ∂B_ε. Let $0<\mu_1(\varepsilon)\le\mu_2(\varepsilon)\le\cdots$ be the eigenvalues of the Laplacian in Ω_ε under the Dirichlet condition on $\gamma\cup\partial B_\varepsilon$. Let $0<\mu_1\le\mu_2\le\cdots$ be the eigenvalues of the Laplacian in Ω under the Dirichlet condition on γ. We arrange them repeatedly according to their multiplicities. Let $\{\varphi_j(\varepsilon)\}_{j=1}^\infty$ (resp. $\{\varphi_j\}_{j=1}^\infty$) be a complete set of orthonormal basis of $L^2(\Omega_\varepsilon)$ (resp. $L^2(\Omega)$) satisfying $-\Delta(\varphi_j(\varepsilon))(x)=\mu_j(\varepsilon)(\varphi_j(\varepsilon))(x)$, $x\in\Omega_\varepsilon$, $(\varphi_j(\varepsilon))(x)=0$ on $\partial\Omega_\varepsilon$ (resp. $-\Delta\varphi_j(x)=\mu_j\varphi_j(x)$, $x\in\Omega$, $\varphi_j(x)=0$ on γ).

We have the following:

Theorem 1. *Fix j. Suppose that μ_j is a simple eigenvalue. Then, the asymptotic relation*

$$\partial(\varphi_j(\varepsilon))(z)/\partial\nu_z^\varepsilon|_{z\in\partial B_\varepsilon}=-\varphi_j(w)\varepsilon^{-1}+O(\varepsilon^{-1/3}) \tag{1}$$

as ε tends to zero. Here $\partial/\partial\nu_z^\varepsilon$ denotes the derivative along the exterior normal direction with respect to Ω_ε.

Remark. Theorem 1 was conjectured in Ozawa [7].

From now on we give a short sketch of our proof of Theorem. We need some lemmas.

Let F be a set in $\boldsymbol{R}^n$. We put

$$|u|_{0,F}=\sup_{x\in F}|u(x)|$$

$$|u|_{\theta,F}=\sup_{x,y\in F}|u(x)-u(y)|/|x-y|^\theta \qquad (0<\theta<1)$$

$$|u|_{1,F}=\sum_{i=1}^n\sup_{x\in F}|\partial_{x_i}u(x)|$$

$$|u|_{2,F}=\sum_{i,j=1}^n\sup_{x\in F}|\partial_{x_i}\partial_{x_j}u(x)|$$

and