47. C^2 Reeb Stability of Noncompact Leaves of Foliations

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1. Introduction. The purpose of this note is to announce a result on the stability of noncompact leaves of codimension one foliations which extends a 2-dimensional theorem of Cantwell-Conlon [1] to all dimensions. We assume throughout that foliations are always transversely orientable, codimension one foliations of closed manifolds with smooth leaves. Recall that a proper leaf of a foliation is *stable* if it admits a trivially foliated, saturated neighborhood (see [3]).

Definition ([1]). A smooth manifold L has the *C^r-stability property* if, whenever L is diffeomorphic to a proper leaf of a C^r foliation, that leaf is stable.

The problem we consider is to characterize the stability property of a manifold L in terms of the topology of L. In the case when L is compact, Thurston [4] has almost completely settled this problem: a compact manifold L has the C^r-stability property ($1 \leqq r \leqq \infty$) if and only if $H^1(L;\boldsymbol{R})=0$. However, in the case when L is noncompact, few partial answers have been known. An important remark is that the direct analogue of Thurston's result does not hold in this case. In fact, it is shown in [1] that there are infinitely many noncompact surfaces with nontrivial real first cohomology groups which have the C^2-stability property. (Although they do not have the C^1-stability property.) Our results give a necessary condition (Proposition 1) and a sufficient condition (Theorem 3) under which a manifold has the C^2-stability property.

2. Statement of results. Let $\hat{H}^1(L;\boldsymbol{R})$ be the image of the canonical homomorphism $H^1_c(L;\boldsymbol{R}) \to H^1(L;\boldsymbol{R})$, where H^1_c denotes the first cohomology group with compact supports. (Note that $\hat{H}^1(L;\boldsymbol{R})$ coincides with $H^1(L;\boldsymbol{R})$ if L is compact.)

First we observe the following

Proposition 1. *Suppose that L is a manifold which can be realized as a proper leaf of some C^r foliation ($0 \leqq r \leqq \infty$). If L has the C^r-stability property, then $\hat{H}^1(L;\boldsymbol{R})=0$.*

This proposition says that the vanishing of $\hat{H}^1(L;\boldsymbol{R})$ is a necessary condition for the stability of L. It is, however, not a sufficient condition. In fact, for example, we have

Proposition 2. *$T^2 \times \boldsymbol{R}$ does not have the C^r-stability property*