18. *On a Pseudo-Runge-Kutta Method of Order 6*

By Masaharu NAKASHIMA
Department of Mathematics, Kagoshima University

(Communicated by Kôsaku YOSIDA, M. J. A., Feb. 12, 1982)

1. Introduction. The present paper is concerned with the numerical solution of the initial value problem:

$$\begin{cases} y' = f(x, y) \\ y(x_0) = y_0. \end{cases} \tag{1.1}$$

In his paper [6], the author has proposed some new method of Runge-Kutta type, and we have seen that there exists an r-stage Pseudo-Runge-Kutta method of order $r+2$ for $r=2, 3$.

Recently he has derived some improvements for this. See [7].

In this paper we shall give a four-stage method of order six based on this improved method. We include also a numerical result to compare our formula with the Huta formula of order 6.

Detailed proofs and related results will appear elsewhere.

2. Numerical method. We consider the Pseudo-Runge-Kutta method, i.e.

$$\begin{aligned} &y_{n+1} = y_n + v(y_n - y_{n-1}) + h\Phi(x_{n-1}, x_n, y_{n-1}, y_n; h) \\ &\Phi(x_{n-1}, x_n, y_{n-1}, y_n; h) = \sum_{i=0}^{4} w_i k_i, \end{aligned} \tag{2.1}$$

where

$$\begin{aligned} &k_0 = f(x_{n-1}, y_{n-1}) \qquad k_1 = f(x_n, y_n), \\ &k_2 = f\left(x_n + a_2 h, y_n + b_0(y_n - y_{n-1}) + h\sum_{i=1}^{2} b_i k_i\right), \\ &k_3 = f\left(x_n + a_3 h, y_n + c_0(y_n - y_{n-1}) + h\sum_{i=1}^{3} c_i k_i\right), \\ &k_4 = f\left(x_n + a_4 h, y_n + d_0(y_n - y_{n-1}) + h\sum_{i=1}^{4} d_i k_i\right). \end{aligned}$$

In the above formula (2.1), the value y_n is to be an approximation to the value $y(x_n)$ of the solution of (1.1) for $x_n = x_0 + nh$.

Throughout the paper, the coefficients $a_i (i=1, 2, 3, 4)$, $b_i (i=0, 1, 2)$, $c_i (i=0, 1, 2, 3)$ and $d_i (i=0, 1, 2, 3, 4)$ are constrained by the following conditions:

$$a_2 = \sum_{i=0}^{2} b_i, \quad a_3 = \sum_{i=0}^{3} c_i, \quad a_4 = \sum_{i=0}^{4} d_i \quad (0 \leqq a_2, a_3, a_4 \leqq 1). \tag{2.2}$$

Assume that $y_n - z(x_n) = O(h^7)$, where $z(x)$ is the solution of the initial value problem $z' = f(x, z)$, $z(x_{n-1}) = y_{n-1}$. Using the same notations as in [6], Taylor expansion for (2.1) is