14. *Analytic Hypo-Ellipticity and Propagation of Regularity for Operators with Non-Involutory Characteristics*

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We study matrices of microdifferential operators of the form $P=P_1P_2I_m+Q$; here P_1 and P_2 are scalar operators such that the Poisson bracket of their principal symbols never vanishes, Q is an $m\times m$ matrix of operators of lower order, and I_m denotes the unit matrix of degree m.

In § 1, we study the propagation of micro-analyticity of solutions of the equation $Pu=0$ when the principal symbol of P_1 is real. Theorem 1 is a partial generalization of Corollary 3.7 of [3], where the principal symbol of P_2 was also assumed to be real.

In § 2, we study the analytic hypo-ellipticity of P when P_1 can be transformed into the form $D_1+\sqrt{-1}x_1^kD_n$ in a neighborhood of $(0, \sqrt{-1}dx_n)\in\sqrt{-1}T^*\boldsymbol{R}^n$ with a positive odd integer k (cf. [5]). Theorem 2 generalizes our previous result (Corollary of [4]) which corresponds to the case $k=1$. To prove Theorem 2, we use different methods from those sketched in [4]; Schapira's theory of positivity (cf. [6]) enables us to reduce the problem of analytic hypo-ellipticity to that of propagation of micro-analyticity of solutions of such equations as treated in § 1.

§ 1. Propagation of regularity. Set $X=\boldsymbol{C}^n\ni z=(z_1, \cdots, z_n)$ and $M=\boldsymbol{R}^n\ni x=(x_1, \cdots, x_n)$. We denote by $T^*X=\{(z,\zeta)\in\boldsymbol{C}^n\times\boldsymbol{C}^n\}$ the cotangent bundle of X, by $T^*_MX=\{(x,\sqrt{-1}\eta)\,;\, x\in\boldsymbol{R}^n,\ \eta\in\boldsymbol{R}^n\}$ the conormal bundle of M in X, and by $\mathcal{C}_M$ the sheaf on T^*_MX of microfunctions. For holomorphic functions f and g defined on an open subset of T^*X, we set

$$H_f=\sum_{j=1}^n\left(\frac{\partial f}{\partial\zeta_j}\frac{\partial}{\partial z_j}-\frac{\partial f}{\partial z_j}\frac{\partial}{\partial\zeta_j}\right)$$

and $\{f,g\}=H_fg$, and denote by f^c the complex conjugate of f with respect to T^*_MX; i.e., f^c is the unique holomorphic function such that $f^c=\bar{f}$ holds on T^*_MX. We denote by σ the principal symbol of a microdifferential operator of finite order, and by σ_j the symbol of order j when the operator is of order at most j.

Let P_1 and P_2 be microdifferential operators of order l_1 and l_2 respectively defined in a neighborhood of $p\in T^*_MX-M$. Set $l=l_1+l_2$ and let $Q=(Q_{ij})$ be an $m\times m$ matrix of microdifferential operators of order