118. Energy Inequality for Non Strictly Hyperbolic Operators in the Gevrey Class

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1. Introduction. In this article, we shall establish the energy inequality for non strictly hyperbolic operators of order m whose coefficients are in the Gevrey class with respect to the space variables and once or twice continuously differentiable with respect to the time variable.

More precisely, we consider the following Cauchy problem,

$$(1.1)\quad \begin{cases} P(y, D_0, D)u = D_0^m u + \sum_{\substack{|\alpha|+j\le m \\ j\le m-1}} a_{\alpha,j}(y) D^\alpha D_0^j u = f & \text{in } G \\ D_0^j u(t_0, x) = u_j(x)\ j=0,1,\cdots,m-1 & \text{in } G_{t_0} = G\cap\{x_0=t_0\}. \end{cases}$$

where $y=(x_0, x)=(x_0, x_1, \cdots, x_d)$,

$$D_0 = \frac{1}{i}\frac{\partial}{\partial x_0}, \qquad D=\left(\frac{1}{i}\frac{\partial}{\partial x_1}, \cdots, \frac{1}{i}\frac{\partial}{\partial x_d}\right),$$

G is a lens of spatial type in R_y^{d+1}.

Let us denote by $P_m(y, \xi_0, \xi)$ the principal symbol of $P(y, D_0, D)$,

$$(1.2)\qquad P_m(y, \xi_0, \xi) = \xi_0^m + \sum_{\substack{|\alpha|+j=m \\ j\le m-1}} a_{\alpha,j}(y)\xi^\alpha \xi_0^j$$

and assume that P is hyperbolic, that is all the ξ_0-roots of $P_m(y, \xi_0, \xi) = 0$ are real and its multiplicity is at most r $(1\le r\le m)$, for any $y\in G$, any $\xi \in R^d\backslash\{0\}$. For an open set G in $R_{x_0}\times R_x^d$, we denote by $\gamma^{K,(s)}(G)$, where $K=0,1,\cdots$, and $1<s<\infty$, the set of functions $f(y)$ such that for any compact set $M\subset G$, there exist constants C, A satisfying the inequalities

$$(1.3)\qquad \sup_{y\in M} |D_0^j D^\alpha f(y)| \le CA^{|\alpha|}(|\alpha|!)^s$$

for all $\alpha\in N^d$, $0\le j\le K$. We also denote $\gamma^{(s)}(\Omega)$, where Ω is an open set in R_x^d, the set of all functions $g(x)$ such that for any compact set $M\subset\Omega$, the following estimates are valid with some constants C, A for all $\alpha\in N^d$

$$(1.4)\qquad \sup_{x\in M} |D^\alpha g(x)| \le CA^{|\alpha|}(|\alpha|!)^s.$$

Recently, M. D. Bronshtein [2] has proved, by constructing the parametrix of which the remainder is a bounded operator in some Hilbert space connected with the Gevrey class, that the problem (1.1) has a unique solution in $\tilde{G}$ $(\subset G)$ for any initial data $u_j(x)\in\gamma^{(s)}(G_{t_0})$, if $a_{\alpha,j}(y)\in\gamma^{K,(s)}(G)$, $K>3(m+d+2)$, $1\le s\le r/(r-1)$. And if $1\le s<r/(r-1)$,