85. *On the Generic Existence of Holomorphic Sections and Complex Analytic Bordism*

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§0. **Introduction.** In this paper, we discuss the generic existence of certain holomorphic sections of holomorphic vector bundles over compact complex manifolds. Moreover we study the relation among Schubert cycles associated with holomorphic sections from a view point of complex analytic bordism.

Let M be a compact complex manifold and $E\to M$ be a holomorphic vector bundle of rank q. For holomorphic sections $\sigma_1, \cdots, \sigma_r$ of $E\to M$, there is an associated Schubert cycle which is defined to be the subset of M where $\sigma_1, \cdots, \sigma_r$ are not linearly independent. Holomorphic sections $\sigma_1, \cdots, \sigma_r$ are said to be in general position if the associated Schubert cycle has codimension $q-r+1$. The bundle $E\to M$ is said to be sufficiently ample if there are global sections $\tau_1, \cdots, \tau_N \in \mathcal{O}(M, E)$ such that (i) the $\tau_i(x)$ span all fibres E_x $(x \in M)$ and that (ii) the differentials $d\tau(x)$ of the sections $\tau=\sum a_i\tau_i (a_i \in \boldsymbol{C})$ which vanish at x span $E_x \otimes \mathcal{T}_x^*$. Cornalba and Griffiths stated in [1] that if the bundle is sufficiently ample, then suitable linear combinations $\sigma_v=\sum a_{vi}\tau_i$ $(v=1, \cdots, r)$ give sections $\sigma_1, \cdots, \sigma_r$ which are in general position. The usefulness of the general position requirement lies in the fact that in this case the associated Schubert cycle represents the Chern class of the given bundle.

One of our results is to show a stronger result by omitting the condition (ii). Moreover it will be shown only under the condition (i) that one can deform arbitrary sections $\sigma_1, \cdots, \sigma_r$ so that the associated Schubert cycle has only singularities of simple type which we call *quasilinear* type. In this case, the deformed sections $\tilde{\sigma}_1, \cdots, \tilde{\sigma}_r$ are said to be in *quasilinear* position. Our generic existence Theorem 3.1 asserts the openness and density of holomorphic sections in quasilinear position.

If two sets of holomorphic sections $\sigma_1, \cdots, \sigma_r$ and $s_1, \cdots, s_r$ are in quasilinear position, then the associated Schubert cycles are homologous, because they represent the same cohomology class. In this paper, we shall show that they are more intimately related. In fact, our bordism theorem shows that there exists a complex analytic bordism of quasilinear type which connect them. The detailed proof