64. Logarithmic Deformations of Holomorphic Maps and Equisingular Displacements of Surfaces with Ordinary Singularities^{*)}

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Introduction. In this paper we shall give a definition of logarithmic deformations of holomorphic maps and prove the existence of the semi-universal family. If the map we consider is non-degenerate, this family turns out to be universal. The concept of logarithmic deformations of holomorphic maps is stimulated by Y. Kawamata's paper [3].

As a by-product we shall obtain another proof of the existence of the universal family of equisingular displacements of surfaces with ordinary singularities, which has been already proved by M. Namba [6]. Following Y. Kawamata, we use the terminology "equisingular" in the sense that they admit a simultaneous *embedded* resolution. Our result cannot cover K. Kodaira's existence theorem [4]. However, in case that the ambient threefold W satisfies the condition $H^2(W, \mathcal{O}_W) = 0$, our theorem includes it.

Our method is expected to be useful for the proof of the higher dimensional analogue of the equisingular displacements of complex spaces with "ordinary singularities".

§1. Logarithmic deformations of holomorphic maps. Let X be a compact complex manifold, C an analytic subset of X of simple normal crossing, and f a holomorphic map of X into a complex manifold Y.

Definition 1. By a family of logarithmic deformations of (X, C, f), we mean a 6-tuple $(\mathcal{X}, C, \phi, \pi, o, T)$ satisfying the following:

(1) $(\mathcal{X}_o, \mathcal{C}_o, \Phi_o) = (X, C, f)$ and $o \in T$,

(2) $(\mathcal{X} - \mathcal{C}, \mathcal{X}, \mathcal{C}, \pi, o, T)$ is a family of logarithmic deformations of $(X - \mathcal{C}, X, \mathcal{C})$ (cf. [3]),

(3) $(\mathfrak{X}, \Phi, \pi, T)$ is a family of holomorphic maps into Y (cf. [5]).

We define the concepts of equivalence and completeness of families of logarithmic deformations of holomorphic maps into Y as in [3] and [5].

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