51. z-Transformation by the New Operator Methods

By Sirō HAYABARA
Okayama University of Science

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§1. **Introduction.** In the theory of electrical engineering or telecommunication engineering, z-transformation technique is widely used for the analysis and synthesis of the sampled data system. The discrete time function which represents the sampled signal or the sampled data system is denoted by

$$f^*(t)=\sum_{n=0}^{\infty} f(nT)\delta(t-nT) \qquad (n=0,1,2,\cdots). \tag{1}$$

The z-transform $F(z)$ of the series $\{f(nT)\}$ is defined as the infinite sum of

$$F(z)=\sum_{n=0}^{\infty} f(nT)z^{-n} \qquad (n=0,1,2,\cdots), \tag{2}$$

where z is the complex variable such as $z=e^{sT}$ for the Laplace variable s. Hence we have

$$F(z)=\sum_{n=0}^{\infty} f(nT)e^{-sTn}=\mathcal{L}f^*(t), \tag{3}$$

and we know that the z-transform for $f(nT)$ is suited to the Laplace transform for $f^*(t)$. We denote (2) as $F(z)=z\{f(nT)\}$.

To obtain the formulae of z-transformation or the inverse z-transformation, several methods such as power series, partial fraction or residues theorems are explained in [4]. However, W. Jentsch [3], and S. Hayabara-S. Haruki [2] have obtained the tables of correspondence for elements of sequence space E, to elements of operator space Q. We call this theory "New Operator Method" (N.O.M.).

In this note, we will prove that the z-transformation is identified with N.O.M. in [2].

Therefore, we will have the following advantages in this direction.

(1) Understanding of the z-transformation is easily gained with the use of fundamental knowledge of the Laplace transformation and the function theory of complex variables.

(2) We can fill up the tables of N.O.M. by using the results of z-transformation which is useful to solve sequence equations or difference equations with variable coefficients.

§2. **z-transformation by the new operator methods.** We have obtained in [2] the operator expression $a_n(p)$ for the sequence $\{a_n\}$ as follows:

$$\{a_n\}=a_n(p)=\sum_{n=0}^{\infty} a_n p^{-n}. \tag{4}$$

By the definition of z-transformation