5. *On Preimage and Range Sets of Meromorphic Functions*[*]

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1. Introduction. We say that two meromorphic functions f and g share a value c provided that $f(z)=c \Longleftrightarrow g(z)=c$ (regardless of multiplicities). In this paper following Gundersen [3] we shall use the abbreviation CM = counting multiplicities and IM = ignoring multiplicities. In [3] it is shown that if two meromorphic functions f and g share three distinct values CM and share a fourth value IM, then they share all four values CM. Several growth relationships between f and g were also established in [3] when they share 3 or 4 values. Actually going back to 1929 the founder of value distribution theory R. Nevanlinna [6] proved that if two nonconstant meromorphic functions f and g share 5 distinct values (possibly including ∞) IM, then f and g must be identical. Thus, the study of the relationship between two meromorphic functions via the preimage sets of several distinct values in the range has a long history. However, only recently, have the studies been extended to include several preimage sets of several disjoint sets of (range) values. The first author of the present paper has already made some contributions on this aspect in [2]. In this paper we shall continue the study and provide some answers to some of the open questions raised in [2], and more importantly we hope that the present results will stimulate additional research and interest in this area.

1.1 Definitions and notations. It is well-known that given any complex number c, every countable discrete set S is the preimage set of c under a certain meromorphic function f. To avoid this trivial case we define a set S to be a nontrivial preimage set (NPS) if S is a countable discrete set and there exists at least one nonlinear entire function f and a finite (range) set T of distinct values with $|T|\geqslant 2$ (where $|T|$ denotes the cardinality of T) such that $f^{-1}(T)=S$. Note that the elements in S need not be distinct. It is natural to ask: does there exist a discrete set S which is not an NPS at all? This can be answered in the affirmative. One can exhibit such sets explicitly according to an argument used in [8]. A generalization of this result

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