38. *Sharpness of Parametrices for Strictly Hyperbolic Operators*

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1. **Introduction.** Let $P(x,D)$ be a linear partial differential operator with C^∞-coefficients defined in R^n and strictly hyperbolic with respect to x_1. Let $\mathrm{E}_k : \mathcal{D}'(Y)\to\mathcal{D}'(\mathrm{R}^n)$ be k-th parametrices, i.e.

$$P(x,D)\mathrm{E}_k\equiv 0, \qquad D_{x_1}^{m-j}\mathrm{E}_k|_{x_1=0}\equiv\delta_{jk}\mathrm{I},$$

where $Y=\{x\in\mathrm{R}^n\,;\,x_1=0\}$ is the initial plane (see e.g. [1]). We want to study the sharpness of distributions $E_k(x,y):=\mathrm{E}_k\delta(x-y)$ here we take $y\in Y$ as parameters. If we take

$\Lambda=\Lambda(y):=\{(x,\xi)\in T^*\mathrm{R}^n\,;\ (x,\xi)$ is on a bicharacteristic strip through some $(y,\eta)\in T^*\mathrm{R}^n$ with $P_m(y,\eta)=0\}$,

$W=W(y):=\pi\Lambda(y)$,

where $\pi: T^*\mathrm{R}^n\to\mathrm{R}^n$ is the natural projection, then we have

$$\operatorname{sing\,supp} E_k(x,y)\subset W(y).$$

Now take a point $x^0\in W$ and a component ω of $\mathrm{R}^n\setminus W$ with $x^0\in\partial\omega$. Then $E_k(x,y)$ is said to be *sharp* at x^0 from ω if there is a neighbourhood V of x^0 and $u\in C^\infty(V)$ such that $E_k(x,y)=u(x)$ on $\omega\cap V$.

Near each point $x^0\in W$, $E_k(x,y)$ can be represented by a finite sum of paired oscillatory integrals $I^\sigma(a,\varphi,x)$, for which L. Gårding [3] discovered a criterion for sharpness. But his arguments and proofs are rather sketchily and, in part, incomplete. Our aim is to clarify the situation and to give a rigorous proofs when $x^0\in W$ is a stable point. Here we use

Definition. $x^0\in W$ is called a stable point if under small perturbations of $\Lambda\subset T^*\mathrm{R}^n$ (as conic Lagrangean manifolds) near $\pi^{-1}(x^0)$, the configurations of W cannot be changed off local diffeomorphisms.

Note that our definition of stability may be considered as a *well posedness* for the problem of sharpness.

If $\pi^{-1}(x^0)\cap\Lambda$ consist of *regular points* (i.e. $N:=\dim T_{\lambda^0}\Lambda\cap T_{\lambda^0}(\text{fibre})=1$ for $\lambda^0\in\pi^{-1}(x^0)\cap\Lambda$), an easy criterion for sharpness are given in [4]. So, in what follows, we shall consider the case when $\pi^{-1}(x^0)\cap\Lambda$ contains *irregular points* (i.e. the case when $N\geqslant 2$).

2. Suppose that $\pi^{-1}(x^0)\cap\Lambda$ consist of stable and irregular points. Then we can prove that, as a germ at x^0, $E_k(x,y)$ can be represented by a finite sum of distributions of the from