17. *Zeta Functions in Several Variables Associated with Prehomogeneous Vector Spaces.* I[*)]

Functional Equations

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1. In this note we introduce zeta functions in several variables associated with prehomogeneous vector spaces defined over the rational number field $\boldsymbol{Q}$ and discuss their functional equations and analytic continuations. Our results are generalizations to those of M. Sato and T. Shintani [4], in which they treated the zeta functions in single variable.

2. Let G be a connected linear algebraic group defined over $\boldsymbol{Q}$. Let ρ_1 and ρ_2 be $\boldsymbol{Q}$-rational representations of G on finite dimensional complex vector spaces E and F with $\boldsymbol{Q}$-structures. Put $\rho=\rho_1\oplus\rho_2$ and $V=E\oplus F$. Here we do not exclude the case where $E=\{0\}$. In the present note we always assume that *(G, ρ, V) is a prehomogeneous vector space* (briefly a p.v.) (for the definition of p.v. and other basic notions in the theory of p.v.'s, we refer to M. Sato and T. Kimura [3]). We assume further that

(A.1) *F is a $\boldsymbol{Q}$-regular subspace of (G, ρ, V)*

in the following sense.

Definition. The invariant subspace F is called a *$\boldsymbol{Q}$-regular subspace of (G, ρ, V)* if there exists a relative invariant $P(x)=P(x^{(1)}, x^{(2)})$ $(x^{(1)}\in E, x^{(2)}\in F)$ of (G, ρ, V) with coefficients in $\boldsymbol{Q}$ such that the Hessian

$$\det\left(\frac{\partial^2 P}{\partial x_i^{(2)}\partial x_j^{(2)}}(x^{(1)}, x^{(2)})\right)$$

of P with respect to the variables $x_1^{(2)}, \cdots, x_{\dim F}^{(2)}$ in F is not identically zero.

Let F^* be the vector space dual to F and ρ_2^* be the representation of G on F^* contragredient to ρ_2. Put $\rho^*=\rho_1\oplus\rho_2^*$ and $V^*=E\oplus F^*$.

Lemma 1. *The triple (G, ρ^*, V^*) is a prehomogeneous vector space and F^* is a $\boldsymbol{Q}$-regular subspace of (G, ρ^*, V^*).*

We call (G, ρ^*, V^*) *the partially dual p.v. of (G, ρ, V) with respect to the $\boldsymbol{Q}$-regular subspace F.*

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