13. *Note on Cyclic Galois Extensions*

By Kozo SUGANO
Department of Mathematics, Hokkaido University

(Communicated by Shokichi IYANAGA, M. J. A., Jan. 12, 1981)

Introduction. As a generalization of central separable algebras, the notion of H-separable extension was introduced in [3]. Especially in the case where B is a commutative ring and A is a faithful B-algebra, A is an H-separable Galois extension of B if and only if A is central Galois extension of B. But in the case where B is non-commutative, there are some properties which hold in H-separable extensions of B but do not hold in central Galois extensions. Especially by Theorem 11 [2] there is no central cyclic Galois extension, while we could find some examples of H-separable cyclic Galois extensions in [11]. The aim of this paper is to show that if A is an H-separable Galois extension of B relative to a cyclic group $G=\langle\sigma\rangle$, then the centralizer of B in A is equal to the center of B (Theorem 1). We will also show that if A is an H-separable extension of B and the center of A is semi-local, then all elements of $\mathrm{Aut}(A|B)$ are inner automorphisms (Theorem 2).

Definitions and symbols. Throughout this paper A will be a ring with the identity 1, B a subring of A which contains 1 of A and C and C' the centers of A and B, respectively. For any subset X of A, any ring automorphism σ of A and any A-A-module M, we will set respectively

$$V_A(X)=\{a\in A\,|\,ax=xa \text{ for all } x \text{ in } X\}$$
$$J_\sigma=\{a\in A\,|\,xa=a\sigma(x) \text{ for all } x \text{ in } A\}$$
$$M^A=\{m\in M\,|\,ma=am \text{ for all } a\in A\}.$$

Furthermore, by A_σ we denote an A-A-module such that $A_\sigma=A$ as left A-module and $ax=a\sigma(x)$ for $a\in A_\sigma$ and $x\in A$ as right A-module. Then we see $J_\sigma=(A_\sigma)^A$, $V_A(B)=A^B=(A_\sigma)^B$, $C=V_A(A)$ and $C'=V_B(B)$. Especially we will denote $D=V_A(B)$. A is an H-separable extension of B if and only if $A\otimes_B A$ is isomorphic to a direct summand of some $(A\oplus A\oplus\cdots\oplus A)$ (finite direct sum) as A-A-module. This condition is equivalent to the condition that for any A-A-module M $D\otimes_C M^A\cong M^B$ by $d\otimes m\to dm$ for $d\in D$ and $m\in M^A$ (see Theorem 1.2 [9]). Hence if A is an H-separable extension of B, $D=(A_\sigma)^B\cong D\otimes_C(A_\sigma)^A=D\otimes_C J_\sigma$. Thus J_σ is rank 1 C-projective, since D is C-finitely generated projective by Theorem 1.1 [3], and $DJ_\sigma=J_\sigma D=D$ for each $\sigma\in\mathrm{Aut}(A|B)$, where $\mathrm{Aut}(A|B)$ denotes the group of all automorphisms of A which fix all elements of B. Furthermore, G will always stand for a finite group of