118. *Connected Sum along the Cycle Operation of* $S^p \times \tilde{S}^{n-p}$ *on π-Manifolds*

By Yoshihiro SAKO
Department of Mathematics, Gakushuin University

(Communicated by Kunihiko KODAIRA, M. J. A., Dec. 12, 1981)

Let M be an n-dimensional simply-connected closed smooth π-manifold. Let p be an integer, $2 \leqq p \leqq (n/2)-2$, and let $\varphi: S^p \to M$ be an imbedding which represents a generator of $H_p(M)$. We consider the operation of the connected sum of M and $S^p \times \tilde{S}^{n-p}$ along the cycle $\varphi(S^p)$ of M and the cycle $S^p \times \{*\}$ of $S^p \times \tilde{S}^{n-p}$, where $\tilde{S}^{n-p}$ denotes a homotopy $(n-p)$-sphere. The topological structure of M is invariant under this operation. We intend to investigate the effect of this operation on the differentiable structure on M.

Let $I_p(M, \varphi)$ denote the set consisting of those homotopy $(n-p)$-spheres which operate on M along the cycle $\varphi(S^p)$ trivially. In this paper we show that $I_p(M, \varphi)$ is trivial in the case where M is a product of standard spheres and φ represents its standard spherical cycle. As an application we show that $S^2 \times S^3 \times \tilde{S}^{10}$ is not diffeomorphic to $S^2 \times S^3 \times S^{10}$, where $\tilde{S}^{10}$ represents a generator of the group of homotopy 10-spheres $\Theta_{10} \approx Z_6$. This result is in contrast with the fact that $S^5 \times \tilde{S}^{10}$ is diffeomorphic to $S^5 \times S^{10}$. Throughout this paper, we mean by a diffeomorphism an orientation preserving diffeomorphism unless otherwise stated.

Details and further arguments will appear elsewhere.

1. Let M be an n-dimensional simply-connected closed smooth π-manifold and $\varphi: S^p \to M$ an imbedding which represents a generator of $H_p(M)$, $p \geqq 2$. Let $\tilde{S}^{n-p} = D_1^{n-p} \bigcup_{\gamma} D_2^{n-p}$ for $\gamma \in \mathrm{Diff}\,(S^{n-p-1})$ and define an imbedding $\iota: S^p \to S^p \times \tilde{S}^{n-p}$ by $\iota(x) = (x, 0)$, where $0 \in D_2^{n-p}$ denotes the center of the $(n-p)$-disk D_2^{n-p}. A trivialization of a tubular neighborhood of $\iota(S^p)$ can be defined by the composition of the maps: $S^p \times D^{n-p} \xrightarrow{1 \times \eta} S^p \times D_2^{n-p} \hookrightarrow S^p \times \tilde{S}^{n-p}$, where $\eta: D^{n-p} \to D_2^{n-p}$ is an orientation reversing diffeomorphism. We denote this trivialization by $I: S^p \times D^{n-p} \to S^p \times \tilde{S}^{n-p}$.

It is known that $S^p \times \tilde{S}^{n-p}$ is diffeomorphic to $S^p \times S^{n-p}$ if $(n-p)-p \leqq 3$. (See Hsiang, Levine and Szczarba [2].) Therefore it suffices to consider the case that $(n-p)-p \geqq 4$, that is, $2 \leqq p \leqq (n/2)-2$.

Since M is a π-manifold, the tubular neighborhood of the imbedded p-dimensional sphere $\varphi(S^p)$ is trivial. We denote this trivialization