

113. Characteristic Indices and Subcharacteristic Indices of Surfaces for Linear Partial Differential Operators

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Let $P(z, \partial_z)$ be a linear partial differential operator with coefficients holomorphic in Ω , $\Omega \subset C^{n+1}$, and $K = \{\varphi(z) = 0\}$ be a nonsingular surface. In the present note we first introduce characteristic indices, subcharacteristic indices and the localization on K of $P(z, \partial_z)$, which represent the relationship between the surface K and $P(z, \partial_z)$. Next we show that they are useful, by considering the equation $P(z, \partial_z)u(z) = f(z)$, where $f(z)$ is holomorphic in $\Omega - K$. The proofs of theorems will be published elsewhere.

§ 1. Definitions. Let C^{n+1} be the $(n+1)$ -dimensional complex space. $z = (z_0, z_1, \dots, z_n) = (z_0, z')$ denotes its point and $\xi = (\xi_0, \xi')$ denotes its dual variable. $\partial_z = (\partial_{z_0}, \partial_{z_1}, \dots, \partial_{z_n}) = (\partial_{z_0}, \partial_{z'})$. For a linear partial differential operator $A(z, \partial_z)$, $A(z, \xi)$ means its total symbol.

Now let us define the localization on K of $P(z, \partial_z)$, characteristic indices σ_i ($1 \leq i \leq p$) and subcharacteristic indices $\sigma_{p,i}$ ($1 \leq i \leq q$). We choose the coordinate so that $\varphi(z) = z_0$. Hence $K = \{z_0 = 0\}$. Let $P(z, \partial_z)$ be a linear partial differential operator of order m in a neighbourhood Ω of $z = 0$. Put

$$(1.1) \quad \begin{cases} P(z, \partial_z) = \sum_{i=0}^m P_i(z, \partial_z) \\ P_i(z, \partial_z) = \sum_{l=0}^i A_{i,l}(z, \partial_{z'}) (\partial_{z_0})^{i-l}, \end{cases}$$

where $A_{i,l}(z, \xi')$ is homogeneous in ξ' , with degree l . We develop $A_{i,l}(z, \xi')$ with respect to z_0 at $z_0 = 0$,

$$(1.2) \quad A_{i,l}(z, \xi') = \sum_{j=0}^{\infty} A_{i,l,j}(z', \xi') (z_0)^j.$$

Let us put

$$(1.3) \quad \begin{cases} d_i = \min \{(l+j); A_{i,l,j}(z', \xi') \not\equiv 0\}. \\ j_i = \min \{j; A_{i,l,j}(z', \xi') \not\equiv 0, l+j = d_i\}. \end{cases}$$

If $A_{i,l}(z, \xi') \equiv 0$ for all l we put $d_i = j_i = +\infty$. We first give

Definition 1.1. The operator $A_{m,L,J}(z', \partial_{z'})$, where $J = j_m$ and $L + J = d_m$, is called the localization on K of $p(z, \partial_z)$.

Let us define characteristic indices σ_i ($1 \leq i \leq p$) which were introduced in Ōuchi [4]. Consider the set $A\{(i, d_i); 0 \leq i \leq m, d_i \neq +\infty\}$ in R^2