109. *A Fundamental Conjecture on Exponent Semigroups*

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1. Introduction. Let S be a semigroup and let N be the multiplicative semigroup of all positive integers. The subset

$$E(S)=\{n \in N \mid (xy)^n = x^n y^n \text{ for all } x, y \in S\}$$

of N forms a subsemigroup of N and is called the *exponent semigroup* of S (Tamura [13]). If $m \in E(S)$ for some $m \geqq 2$, we say S is an *E-m semigroup*. The structure of E-m semigroups has been studied by Nordahl [11] and Cherubini and Varisco [2], and the structure of E-m groups was described by Alperin [1]. However, the structure of $E(S)$ itself had been veiled until a recent date. Only recently, Clarke, Pfiefer and Tamura [4] proved that if $2 \in E(S)$, $E(S)$ is equal to either N or $N \backslash \{3\}$. Inspired by their work, Kobayashi [8] studied the case $3 \in E(S)$ and has determined the structure of such $E(S)$ up to modulo 6.

Suggested by the results on the case $3 \in E(S)$, we present in this note a fundamental conjecture which describes the structure of the exponent semigroups containing m modulo $m(m-1)$. We give several results supporting the validity of the conjecture. Above all, the conjecture is true for all finite semigroups. Most of the results will be given in [9] with complete proofs.

2. Reduction to mod $m(m-1)$. The following two theorems tell us that in studying the structure of $E(S)$ for an E-m semigroup S, it is essential to consider it modulo $m(m-1)$.

Theorem 1. *Let S be an E-m semigroup. If $k \in E(S)$ for some integer $k \geqq m$, then $\alpha m(m-1)+k \in E(S)$ for all integers $\alpha \geqq 0$.*

Corollary (Cherubini and Varisco [2]). *If S is an E-m semigroup, then $\alpha m(m-1)+m \in E(S)$ for all integers $\alpha \geqq 0$.*

Theorem 2. *Let S be an E-m semigroup. If $k \in E(S)$ for some integer $k \geqq 1$, then $\alpha m(m-1)+k \in E(S)$ for all integers $\alpha \geqq 2$.*

Corollary (cf. [8, Lemma 5]). *If S is an E-m semigroup, then $\alpha m(m-1)+1 \in E(S)$ for all integers $\alpha \geqq 2$.*

It has been shown by Tamura [14] that for any integer $m \geqq 2$, there is an E-m semigroup S such that $E(S)$ does not contain $m(m-1)+1$. However, the following problem is still open: *For any E-m semigroup S, does $k(\geqq 2) \in E(S)$ imply $m(m-1)+k \in E(S)$?* When $m \leqq 3$, the answer is positive by [4] and [8], and when $k \geqq m$, Theorem 1 gives a positive answer. We can prove that the problem has an af-