108. *Fourier Transforms of Nilpotently Supported Invariant Functions on a Finite Simple Lie Algebra**)

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0. Let $\mathfrak{G}$ be a connected simple algebraic group defined over a finite field $k=\boldsymbol{F}_q$, and let $\mathfrak{g}=\mathrm{Lie}(\mathfrak{G})$, the Lie algebra of $\mathfrak{G}$. We denote by σ the Frobenius morphism, and by G (resp. g) the set $\mathfrak{G}_\sigma$ (resp. $\mathfrak{g}_\sigma$) of σ-fixed points of $\mathfrak{G}$ (resp. $\mathfrak{g}$). Let $\mathrm{Inv}(g)$ be the space of $\boldsymbol{C}$-valued $\mathrm{Ad}(G)$-invariant functions on g and $\mathrm{Inv}(g_0)$ the subspace of $\mathrm{Inv}(g)$ consisting of all $f\in\mathrm{Inv}(g)$ supported by the set g_0 of nilpotent elements of g. In § 2, we introduce an operation $f\to f^\wedge$ for $f\in\mathrm{Inv}(g)$, and in § 3, we define the 'Fourier transform' $\mathcal{F}(f)$ for $f\in\mathrm{Inv}(g_0)$. The main result (Theorem 3) of this paper says that these two operations coincide with each other on a relatively large subspace $\mathrm{Inv}(g_0)'$ of $\mathrm{Inv}(g_0)$, if the characteristic of k is not too small. As a corollary, we can prove orthogonality relations (Cor. 2) for $\{\mathcal{F}(1_{O_\sigma})\}_O$, where O runs over the set of σ-stable nilpotent $\mathrm{Ad}(\mathfrak{G})$-orbits in $\mathfrak{g}$ and 1_{O_σ} is the characteristic function of O_σ. This can be considered as a counterpart to a result [7, 5.6] of T. A. Springer. (He treated the case of strongly regular (semisimple) orbits rather than nilpotent orbits.) At the end of the paper we present a curious fact (Theorem 4) on the distribution of nilpotent elements in g. Although this result is not directly related to our main results, Theorem 4 and Corollaries 1, 2 show that the variety $\mathfrak{g}_0$ of nilpotent elements of $\mathfrak{g}$ sometimes looks like a $2N$-dimensional vector subspace of $\mathfrak{g}$, where $2N=\dim\mathfrak{g}_0$.

Details and proofs are omitted and will be published elsewhere.

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1. Let $\mathfrak{B}$ be a σ-stable Borel subgroup of $\mathfrak{G}$ and $\mathfrak{T}$ a σ-stable maximal torus contained in $\mathfrak{B}$. Put $B=\mathfrak{B}_\sigma$ and $N(\mathfrak{T})=$ the normalizer of $\mathfrak{T}$ in $\mathfrak{G}$. Then $(G,B,N(\mathfrak{T})_\sigma)$ is a Tits system with the Weyl group $W=N(\mathfrak{T})_\sigma/\mathfrak{T}_\sigma$. Let (W,R) be the associated Coxeter system. Then, to each $J\subset R$, there corresponds a σ-stable parabolic subgroup $\mathfrak{P}_J$ of

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