107. On Fourier Coefficients of Klingen's Eisenstein Series of Degree Three

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In this note, we shall give explicit formulas for some Fourier coefficients of Klingen's Eisenstein series of degree three. The author would like to thank Prof. Kitaoka for his helpful advices.

Let H_n denote the Siegel upper half-space of $n\times n$ symmetric complex matrices with positive definite imaginary part. Put $\Gamma_n=\mathrm{Sp}(n,\boldsymbol{Z})$. Let M_n^k denote the space of Siegel modular forms of degree n and of weight k with respect to Γ_n. Let $f\in M_r^k$ be a cusp form on H_r. For even k, $k>n+r+1$ and $n>r$, the Eisenstein series $E_{n,r}^k(Z,f)$ is defined, following Klingen [5], by the following series:

$$E_{n,r}^k(Z,f)=\sum_{M\in\Delta_{n,r}\backslash\Gamma_n} f(M\langle Z\rangle^*)|CZ+D|^{-k}$$

and its Fourier expansion is denoted by

$$=\sum_{T\geq 0} a(T;[f])\exp(2\pi i\,\mathrm{Tr}\,(TZ)).$$

In [3], Harris proved that $E_{n,r}^k(Z,f)$ has algebraic Fourier coefficients whenever f does without showing any explicit formula for $a(T;[f])$ (see also [6]). On the other hand, Mizumoto [8] proved explicit formulas for some Fourier coefficients of $E_{2,1}^k(Z,f)$. Here we shall study Fourier coefficients of $E_{3,2}^k(Z,f)$ using a different idea from Mizumoto's, suggested by Kitaoka.

To state our result precisely, let $f\in M_2^k$ be a cusp form which is a common eigenfunction for all Hecke operators $T(m)$. We denote its Fourier expansion by

$$f(z)=\sum_{N\geq 0} b(N)\exp(2\pi i\,\mathrm{Tr}\,(NZ)),\qquad Z\in H_2.$$

Let $Z_f^{(2)}(s)$ denote the symmetric square of the zeta-function corresponding to f (see Def. 2.1. in [1]). Let T be any 3×3 positive definite semi-integral matrix. Put $\Delta(T)=|2T|$. For such T, we associate an analytic class invariant $\vartheta_T(Z)$, $Z\in H_2$, by the following series:

$$\vartheta_T(Z)=\sum_{M\in M_{2\times 3}(\boldsymbol{Z})}\exp(2\pi i\,\mathrm{Tr}\,(MT{}^tMZ)).$$
$$=\sum_{N\geq 0} c(N)\exp(2\pi i\,\mathrm{Tr}\,(NZ)).$$

For any Dirichlet character χ, we define a Dirichlet series $D(s,f,\vartheta_T,\chi)$ by the following series like in [7]:

$$D(s,f,\vartheta_T,\chi)=\sum_{\{N\}}\frac{b(N)c(N)\chi(|2N|)}{\varepsilon(N)|N|^s}.$$