106. On the Attractivity Properties for the Equation $x''+a(t)f_1(x)g_1(x')x'+b(t)f_2(x)g_2(x')x=e(t, x, x')$

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(Communicated by Kôsaku YOSIDA, M. J. A., Nov. 12, 1981)

1. Introduction. In this paper we shall study the asymptotic behavior of solutions of the second order differential equation

(1) $$x''+a(t)f_1(x)g_1(x')x'+b(t)f_2(x)g_2(x')x=e(t, x, x')$$

or an equivalent system

(2) $$x'=y, \qquad y'=-a(t)f_1(x)g_1(y)y-b(t)f_2(x)g_2(y)x+e(t, x, y),$$

where $a(t)>0$, $b(t)>0$, $f_i(x)>0$ and $g_i(y)>0$ $(i=1, 2)$.

In [1], the following theorem was given by T. A. Burton for the system

(3) $$x'=y, \qquad y'=-p(x)\,|y|^{\alpha}y-g(x),$$

where $p(x)>0$ and $0\leqq\alpha<1$.

Theorem (Burton). *The zero solution of* (3) *is globally asymptotically stable if and only if* $\int_0^{\pm\infty}[p(x)+|g(x)|]dx=\pm\infty$.

In [2], Burton had an extension of this theorem for the following system:

(4) $$x'=y, \qquad y'=-f(x)h(y)y-g(x)+e(t).$$

On the other hand, for the system

(5) $$x'=y, \qquad y'=-f(x)h(y)y-g(x)k(y)+e(t),$$

J. W. Heidel proved in [3] that if $\int_0^{\pm\infty}[f(x)+|g(x)|]dx=\pm\infty$ and if $k(y)$ satisfies some conditions, then all solutions of (5) converge to the origin as $t\to\infty$, that is the origin is attractive for (5).

The purpose of this paper is to give a sufficient condition and a necessary condition for the convergence of all solutions of (2) to the origin as $t\to\infty$ under the following assumptions.

(I) *$a(t)$ and $b(t)$ are continuously differentiable in* $[0, \infty)$.

(II) *$f_1(x)$, $f_2(x)$, $g_1(y)$ and $g_2(y)$ are continuous in R^1 and $e(t, x, y)$ is continuous in* $[0, \infty)\times R^2$.

(III) $$\int_0^{\infty}\frac{|a'(t)|}{a(t)}dt<\infty \quad \textit{and} \quad \int_0^{\infty}\frac{|b'(t)|}{b(t)}dt<\infty.$$

(IV) $$\int_0^{y}\frac{v}{g_2(v)}dv\to\infty \quad \textit{as} \quad |y|\to\infty.$$

(V) $$\frac{y^2}{g_2(y)}\leqq M\int_0^{y}\frac{v}{g_2(v)}dv \qquad \textit{for } y\in R^1, \textit{ where } M>0.$$