104. *Degeneration of the Two Dimensional Garnier System*

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(Communicated by Kôsaku YOSIDA, M. J. A., Nov. 12, 1981)

1. Introduction. In 1907, R. Fuchs [1] found out a remarkable connection between the sixth Painlevé equation and a second order linear ordinary differential equation

$$\frac{d^2y}{dx^2}=p(x;t)y. \tag{1.1}$$

He showed that the sixth Painlevé equation is just a deformation equation of a linear equation (1.1) of Fuchsian type, where t is a deformation parameter. The above result was extended by R. Garnier [2] in two directions. Firstly, he showed that the other five equations of Painlevé can be obtained from the isomonodromic deformation of linear equations with irregular singular points of the form (1.1). Secondly, he derived a completely integrable system of nonlinear Pfaffian equations from the isomonodromic deformation of (1.1) with N deformation parameters $t=(t_1, \cdots, t_N)$. This system can be considered as a generalization of the sixth Painlevé equation. Recently K. Okamoto [3] obtained a Hamiltonian system by considering the isomonodromic deformation of the following equation of Fuchsian type with N deformation parameters $t=(t_1, \cdots, t_N)$

$$\frac{d^2y}{dx^2}+p_1(x;t)\frac{dy}{dx}+p_2(x;t)y=0, \tag{1.2}$$

and he showed that Garnier's system can be transformed into a Hamiltonian system. By *N dimensional Garnier system* we mean the Hamiltonian system obtained by K. Okamoto from the isomonodromic deformation of (1.2) and it will be denoted by G_N.

The purpose of this note is to derive Hamiltonian systems from the two dimensional Garnier system G_2 by making a process of step-by-step degeneration and to show that this process of degeneration is induced by a process of confluence of singularities of linear equation (1.2).

2. Confluence of singularities. We consider the equation (1.2) with $t=(t_1, t_2)$ and we start from the equation $\mathrm{L}_{\mathrm{VIII}}$ given by

$$\mathrm{L}_{\mathrm{VIII}} \quad p_1(x;t)=\frac{1-\kappa_0}{x}+\frac{1-\kappa_1}{x-1}+\sum_j\frac{1-\theta_j}{x-t_j}-\sum_j\frac{1}{x-\lambda_j},$$

$$p_2(x;t)=\frac{\kappa}{x(x-1)}-\sum_j\frac{t_j(t_j-1)H_j}{x(x-1)(x-t_j)}+\sum_j\frac{\lambda_j(\lambda_j-1)\mu_j}{x(x-1)(x-\lambda_j)},$$