101. *Calculus on Gaussian White Noise. III*

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In the previous parts of this series [11], [12], we have given a systematic treatment of calculus on Gaussian white noise, which is a reformulation of Hida's works [1], [2]. In this part we will show further relations between Hida's approach and ours. We will use the same notations and definitions as in Part I and Part II.

§8. Multiple Wiener integrals. Here we assume that the Borel measure ν on T has no atoms. Let $\mathcal{E}\subset E_0=L^2(T,\nu)\subset\mathcal{E}^*$ be a triplet as in §5 of Part II, and let μ be the measure of Gaussian white noise on $\mathcal{E}^*$ with characteristic functional $\exp[-\|\xi\|_0^2/2]$. The multiple Wiener integral $I_n(F_n)$ of F_n in $L^2(T^n,\nu^n)$ is defined as follows:

First, $I_1(F_1)$ is the limit of $\langle x,\xi_k\rangle$ in $(L^2)=L^2(\mathcal{E}^*,\mu)$, where $\{\xi_k\}$ is any sequence in $\mathcal{E}$ with $\|\xi_k-F_1\|_0\to 0$, as $k\to\infty$. Specially, put $W(B)\equiv I_1(I_B)$, where I_B denotes the indicator function of a Borel set B with $\nu(B)<\infty$. Secondary, let $\alpha=\{B_j\}$ be a countable Borel partition of T with $\nu(B_j)<\infty$ and let α^n be the collection of all subsets of T^n of the form $C=B_{j(1)}\times B_{j(2)}\times\cdots\times B_{j(n)}$, $B_{j(k)}\in\alpha$, $B_{j(k)}\cap B_{j(m)}=\phi$ for $k\neq m$. For such a set C in α^n, define

$$I_n(I_C)\equiv\prod_{k=1}^{n} W(B_{j(k)}).$$

Define $I_n(G_n)\equiv\sum a_kI_n(I_{C_k})$ for $G_n=\sum a_kI_{C_k}$ with $C_k\in\alpha^n$. Then we can define $I_n(F_n)$ by

$$(8.1)\qquad I_n(F_n)\equiv\underset{\alpha\uparrow}{\mathrm{l.i.m.}}\, I_n(F_n^\alpha),\qquad F_n^\alpha\equiv\sum\nu^{-1}(C)(F_n,I_C)I_C,$$

where $\alpha\uparrow$ means refinements.

Theorem 8.1. (i) *For $F_n\in L^2(T^n,\nu^n)$, put $\varphi(x)=I_n(F_n)$, then we have*

$$(\mathcal{S}\varphi)(\xi)=\int_{T^n}F_n(u_1,\cdots,u_n)\xi(u_1)\cdots\xi(u_n)d\nu^n(u_1,\cdots,u_n).$$

(ii) *For any ψ in (L^2), there exist $F_n\in\hat{L}^2(T^n,\nu^n)$, $n\geq 0$, such that $\psi(x)$ is decomposed into the following orthogonal sum;*

$$\psi(x)=\sum_{n=0}^{\infty}I_n(F_n)\quad\textit{and}\quad\|\psi\|_{(L^2)}^2=\sum_{n=0}^{\infty}n!\,\|F_n\|_{\hat{L}^2(T^n,\nu^n)}^2.$$

We now remark that the symmetric L^2-space $\hat{L}^2(T^n,\nu^n)$ is naturally identified with the symmetric tensor product space $E_0^{\hat{\otimes}n}$. By Theorems 6.3 and 6.5, we have