91. *Singular Cauchy Problems for a Class of Weakly Hyperbolic Differential Operators*

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In these notes singular Cauchy problems of Hamada's type are studied in the category of holomorphic functions and hyperfunctions for a class of hyperbolic differential operators with non-involutive multiple characteristics. Integral representations of their solutions are given.

1. Introduction. Let $P(t, x, D_t, D_x)$ be a differential operator of order m of the form

$$P(t, x, D_t, D_x)=D_t^m+\sum_{i=1}^m A_i(t, x, D_x)D_t^{m-i},$$

where $D_t=(1/\sqrt{-1})(\partial/\partial t)$, $D_x=(1/\sqrt{-1})(\partial/\partial x)$ and $A_i(t, x, D_x)$ is a differential operator at most of order i, not containing D_t, whose coefficients are holomorphic functions defined in a neighborhood of $(t, x)=(0, 0)$ in $\boldsymbol{C}\times\boldsymbol{C}^n$.

We assume the following conditions:

(A-1) (Degeneracy of characteristic roots). There exists a non-negative integer q such that the principal symbol $P_m(t, x, \tau, \xi)$ of $P(t, x, D_t, D_x)$ is expressed in the form

$$P_m(t, x, \tau, \xi)=\prod_{j=1}^m (\tau-t^q\lambda_j(\xi)),$$

where $\lambda_j(\xi)$ $(1\leq j\leq m)$ are holomorphic functions defined in a conic open neighborhood Ω_0 of $\xi_0=(1, 0, \cdots, 0)$ in $\boldsymbol{C}^n-0$ and homogeneous of degree 1 such that

$$\lambda_j(\xi)\neq\lambda_k(\xi), \qquad \text{if } j\neq k \text{ and } \xi\in\Omega_0.$$

(A-2) (Hyperbolicity). $\lambda_j(\xi)$ $(1\leq j\leq m)$ are real if ξ is real.

(A-3) (Levi condition). Let $A_{i,j}(t, x, \xi)$ be the homogeneous part of $A_i(t, x, \xi)$ of degree j with respect to ξ and let

$$A_{i,j}(t, x, \xi)=\sum_{k=0}^\infty t^k A_{i,j,k}(x, \xi)$$

be the Taylor expansion of $A_{i,j}(t, x, \xi)$ with respect to t. Then

$$A_{i,j,k}(x, \xi)=0, \qquad \text{if } k<(q+1)j-i.$$

Alinhac [1], Amano [2], Amano-Nakamura [13], Nakamura-Uryu [6], Nakane [7], Taniguchi-Tozaki [10] and Yoshikawa [12] studied the Cauchy problem for weakly hyperbolic operators of the above type, and constructed parametrices, using a type of ordinary differential operators with polynomial coefficients which determine the principal