# 87. *Modular Forms of Degree n and Representation by Quadratic Forms. III*

## *Kloosterman's Method*

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Kloosterman improved a result of Hecke about estimates of Fourier coefficients of cusp forms by using so-called Kloosterman sums. Our aim is to generalize his method to Siegel modular forms of degree 2 with two assumptions on exponential sums and to apply it to representations by quadratic forms.

**Terminology and notations.** Let $H$ be the space of $2\times 2$ complex symmetric matrices $Z$ whose imaginary part is positive definite, and $\Gamma=Sp_2(\boldsymbol{Z})$ which acts on $H$ discontinuously. Denote by $\mathfrak{F}$ the fundamental domain $\Gamma\backslash H$ by Siegel (p. 169 in [5]). By $\Gamma(\infty)$ we denote the subgroup $\left\{\begin{pmatrix} * & * \\ 0 & * \end{pmatrix}\in\Gamma\right\}$ of $\Gamma$ where 0 is the $2\times 2$ zero matrix and $\mathfrak{G}$ stands for $\bigcup_{M\in\Gamma(\infty)} M\langle\mathfrak{F}\rangle$. By $\Lambda, \boldsymbol{Q}\Lambda$ and $\boldsymbol{R}\Lambda$ we denote the set of integral, rational and real symmetric $2\times 2$ matrices respectively, and $\Lambda^*$ stands for $\{(s_{ij})\in\boldsymbol{Q}\Lambda \mid s_{11}, s_{22}\in\boldsymbol{Z}, 2s_{12}\in\boldsymbol{Z}\}$. For $C, D\in M_2(\boldsymbol{Z})$, $(C, D)=1$ means that there exist matrices $A, B\in M_2(\boldsymbol{Z})$ such that $\begin{pmatrix} A & B \\ C & D\end{pmatrix}\in\Gamma$. $\sigma$ stands for the trace of square matrices and $e(z)$ means $\exp(2\pi iz)$ for a complex number $z$.

We gather two assumptions and some lemmas.

**Assumption 1.** *Let $c_1, c_2$ be natural numbers with $c_1\mid c_2$ and $Y\in\boldsymbol{R}\Lambda$ positive definite. Then we assume*

$$\sum_{g_i}\Big|\sum_{s_i} e(s_1g_1/c_1+s_2g_2/c_1+s_4g_4/c_2)\Big|=O(c_1^2c_2^{1+\varepsilon}) \qquad \textit{for any } \varepsilon>0,$$

*where $g_1, g_2, s_1, s_2$ run over $\boldsymbol{Z}/c_1\boldsymbol{Z}$ and $g_4, s_4$ run over $\boldsymbol{Z}/c_2\boldsymbol{Z}$ and moreover $\{s_i\}$ satisfies*

$$\begin{pmatrix} s_1/c_1 & s_2/c_1 \\ s_2/c_1 & s_4/c_2\end{pmatrix}+\sqrt{-1}Y\in\mathfrak{G}.$$

*Here $O$ is independent of $Y$.*

**Assumption 2.** *Let $C\in M_2(\boldsymbol{Z})$, $|C|\neq 0$. For $G_1, G_2\in\Lambda^*$ we put*

$$K(G_1, G_2; C)=\sum_D e(\sigma(AC^{-1}G_1+C^{-1}DG_2))$$

*where $D$ runs over $\{D\in M_2(\boldsymbol{Z}) \bmod C\Lambda \mid (C, D)=1\}$ and $A$ is a matrix such that $\begin{pmatrix} A & * \\ C & D\end{pmatrix}\in\Gamma$. For these generalized Kloosterman sums we*