74. On Eisenstein Series for Siegel Modular Groups. II

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Introduction. This is a continuation of [11]. We note the Fourier coefficients of Eisenstein series concerning the following two points: the rationality and an interpretation of the explicit formula. The author would like to thank Profs. M. Harris, T. Oda, and D. Zagier for kindly communicating their preprints: [4], [5], [18], [20]. (This paper was revised into the present form in March-April 1981 when the author received their preprints; the original preprint was cited in [9-II], [12].) We follow the previous notations of [8]–[12].

§1. **L-functions.** We fix our notation on two L-functions attached to Siegel eigenforms. Let f be a Siegel eigen modular form in $M_k(\Gamma_n)$ for integers $n\geqq 0$ and $k\geqq 0$. We denote by $L_1(s,f)$ the first L-function attached to f (an Euler product over $\boldsymbol{Q}$ of degree 2^n), which is defined in Andrianov [1]. We put $L_1^u(s,f)=L_1(s+n(2k-n-1)/4,f)$. If $n=0$ then $L_1(s,f)=L_1^u(s,f)=\zeta(s)$ (cf. [10, §3]). It is expected that $L_1^u(s,f)$ is meromorphic on $\boldsymbol{C}$ with functional equation for $s\to 1-s$. This is known for $n\leqq 2$. A formulation of Ramanujan conjecture for f is that $L_1^u(s,f)$ is unitary in the sense of [7]; cf. [8, p. 150, p. 165]. We denote by $L_2^u(s,f)$ the second L-function attached to f (an Euler product over $\boldsymbol{Q}$ of degree $2n+1$), which is defined in Andrianov [2]. If $n=0$ then $L_2^u(s,f)=\zeta(s)$. It is expected that $L_2^u(s,f)$ is meromorphic on $\boldsymbol{C}$ with functional equation for $s\to 1-s$. This is proved in certain cases by Shimura [19] and Andrianov-Kalinin [3]. For $n=1$ we have $L_2^u(s,f)=L_2(s+k-1,f)$ in the previous notation.

We note relations between L-functions for two liftings.

(A) For each eigen modular form f in $M_k(\Gamma_1)$ we have

$$L_1^u(s,[f])=L_1^u(s+(k-2)/2,f)L_1^u(s-(k-2)/2,f)\quad\text{and}$$
$$L_2^u(s,[f])=L_2^u(s,f)\zeta(s+k-2)\zeta(s-k+2).$$

More generally let F be an eigen modular form in $M_k(\Gamma_n)$ such that $\Phi(F)\neq 0$. Then we have

$$L_1^u(s,F)=L_1^u(s+(k-n)/2,\Phi(F))L_1^u(s-(k-n)/2,\Phi(F))\quad\text{and}$$
$$L_2^u(s,F)=L_2^u(s,\Phi(F))\zeta(s+k-n)\zeta(s-k+n).$$

(B) For each eigen modular form f in $M_{2k-2}(\Gamma_1)$ we have

$$L_1^u(s,\sigma_k(f))=L_1^u(s,f)\zeta(s+1/2)\zeta(s-1/2)\quad\text{and}$$
$$L_2^u(s,\sigma_k(f))=L_1^u(s+1/2,f)L_1^u(s-1/2,f)\zeta(s).$$

§2. **Fourier coefficients.** For a modular form f in $M_k(\Gamma_n)$ ($n\geqq 0$