72. *On Asymptotic Equivalence of Bounded Solutions of Two Integro-Differential Equations*

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Abstract. In this note we consider some problems concerning the asymptotic equivalence of bounded solutions of integro-differential equations.

Consider the perturbed system of integro-differential equations

$$\text{(P)} \qquad x'(t)=A(t)x(t)+\int_{t_0}^{t} B(t,s)x(s)ds+f(x)(t), \qquad t\geqq t_0,$$

where A, B are given $n\times n$ matrices and the perturbation n vector $f(x)(\,.\,)$ is an operator mapping the set of functions defined for $t\geqq t_0$ into itself; for example, typical perturbations are of the form

$$f(x)(t)=f(t,x(t)) \quad \text{or} \quad \int_{t_0}^{t} K(t,s,x(s))ds$$
$$\text{or} \quad x(t)\int_{t_0}^{t} K(t,s,x(s))ds.$$

We are interested in comparing the bounded solutions of (P) with those of the related unperturbed linear system

$$\text{(L)} \qquad y'(t)=A(t)y(t)+\int_{t_0}^{t} B(t,s)y(s)ds, \qquad t\geqq t_0.$$

In particular, we will determine conditions on A, B and f so that each bounded solution y of (L) corresponds to a bounded solution x of (P), in such a way that their difference $y-x$ tends to zero asymptotically, and conversely, each bounded solution x of (P) corresponds to a bounded solution y of (L) such that their difference $x-y$ tends again to zero asymptotically. In other words, the systems (P) and (L) should be asymptotically equivalent.

J. A. Nohel ([7], [8]) has established the asymptotic equivalence of (P) and (L) in the case that the linear system (L) is asymptotically stable. Our aim is to cover the cases that the linear system (L) is conditionally asymptotically stable, conditionally uniformly asymptotically stable and conditionally uniformly stable.

The fundamental solution matrix (or resolvent kernel) of (L) is the solution $Y(t,s)$ of the matrix equation

$$\frac{\partial}{\partial t}Y(t,s)=A(t)Y(t,s)+\int_{s}^{t} B(t,r)Y(r,s)dr, \qquad t\geqq s\geqq t_0,$$
$$Y(s,s)=I,$$