67. On the Isomonodromic Deformation for Linear Ordinary Differential Equations of the Second Order

By Tosihusa KIMURA
Department of Mathematics, University of Tokyo

(Communicated by Kôsaku YOSIDA, M. J. A., June 11, 1981)

§ 1. **Introduction.** It was R. Fuchs ([1]) who gave first an example of the isomonodromic deformation. Considering the differential equation of Fuchsian type

$$\frac{d^2y}{dx^2}=\left(\frac{a}{x^2}+\frac{b}{(x-1)^2}+\frac{c}{(x-t)^2}+\frac{3}{4(x-\lambda)^2}\right.$$
$$\left.+\frac{\rho}{x}+\frac{\sigma}{x-1}+\frac{\tau}{x-t}+\frac{\omega}{x-\lambda}\right)y$$

having $x=\lambda$ as apparent singularity, he rediscovered the sixth Painlevé equation as isomonodromic deformation equation. Then R. Garnier ([2]) derived all the other Painlevé equations by the isomonodromic deformation for linear differential equations of the form

$$y''=py \tag{1.1}$$

with irregular singularities and an apparent singularity. (For the isomonodromic deformation of equations with irregular singularities, see [3], [4], [7].)

Recently K. Okamoto ([5], [6]) found the following two remarkable facts: 1) The Painlevé equations are converted into Hamiltonian systems, called the Painlevé systems, with polynomial Hamiltonian functions. 2) If the linear differential equations considered by Fuchs and Garnier are transformed into equations of the form

$$y''+p_1y'+p_2y=0 \tag{1.2}$$

in a canonical way, then the isomonodromic deformation for the transformed equations is governed by the Painlevé systems.

Fuchs and Garnier, and hence Okamoto supposed that the difference of the exponents at the apparent singularity is just two up to the sign. The purpose of this note is to discuss the case when this difference is greater than two.

§ 2. **Preliminaries.** If the equation (1.2) is transformed into an equation of the form (1.1), we have

$$p=\frac{1}{2}p_1'+\frac{1}{4}p_1^2-p_2. \tag{2.1}$$

Suppose that p_1 and p_2 are rational in x and in several parameters.