5. *Bochner Problem on a Topological Vector Space with a Quasi-Invariant Measure*

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(Communicated by Kôsaku YOSIDA, M. J. A., Jan. 12, 1981)

1. Let F be a locally convex space and ρ be another vector topology on F. Then ρ is called *admissible* if for every positive definite function ϕ, the ρ-continuity of ϕ is equivalent to the existence of a $\sigma(F', F)$-Radon measure on F' (the topological dual) with the Fourier transform ϕ. The admissible topologies are not unique even on the Hilbert space (cf., Sazonov [9] and Gross [1]). We say the weakest of all admissible topologies (if exists) *the S-topology*. In case of a Hilbert space and a nuclear Fréchet space, the admissible topologies were given by Sazonov [9] and Minlos [6], respectively. Mouchtari [7] proved that if F is a Banach space with the metric approximation property and F' is L^0-embeddable, then the S-topology exists, and gave explicitly the S-topology on L^p $(2\leqq p<\infty)$. The first aim of this paper is to prove that if F is a Banach space of dual $\mathcal{S}_p$ $(1<p\leqq 2)$, then the S-topology exists and is given by a family of absolutely summing operators (Theorem 1).

Let ϕ be a continuous positive definite function on a locally convex space F. Then, in general, a σ-additive measure with the Fourier transform ϕ does not exist on F'. Sazonov [9] proved that, in the case where F is a Hilbert space, a σ-additive measure with the Fourier transform ϕ exists on a Hilbert-Schmidt extension of F'. The second aim of this paper is to find a suitable extension G of F' such that for every ϕ, there exists a σ-additive measure on G with the Fourier transform ϕ.

Let E be a locally convex space of second category (or barrelled), F be a complete locally convex space with the metric approximation property and $\iota : E\to F$ be a continuous injection. We shall show that if there exists an $\iota(E)$-quasi-invariant Radon measure on F, then for every continuous positive definite function ϕ on F, a $\sigma(E', E)$-Radon measure with the Fourier transform $\phi\circ\iota$ exists on E' (Theorem 3). The fundamental tool to prove Theorem 3 is a generalization of Xia's inequality (Xia [11], also Koshi and Takahashi [1]).

2. Let $FD(F)$ be the set of all finite-dimensional subspaces of F and $W^0=\{x'\in F' ; \langle x, x'\rangle=0$ for every $x\in W\}$ for $W\in FD(F)$. The finite-dimensional space F'/W^0 has the natural Borel field $\mathcal{B}(W)$. We