# 58. On the Solvability of Nonlinear Goursat Problems

By Masafumi YOSHINO
Department of Mathematics, Tokyo Metropolitan University

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1. **Introduction.** In the study of Cauchy-Goursat problems for nonlinear analytic equations, the spectral condition, that is the condition of the minimum radius for the exceptional set of a parameter, has been a well accepted starting point (cf. [1]). Outside this exceptional disk, the problems have unique analytic solution, but no general results are known as to the geometric structure of the exceptional set inside the disk or its relation to the equation itself. In this paper, we shall give an existence-uniqueness theorem inside the disk with the description of the exceptional set, together with an alternative theorem for Goursat problems as an immediate consequence.

The theorem is proved by applying the contraction principle on some suitable Banach space. But the essential point of the arguments here lies in the proof of the existence and continuity of the inverse of the linearized operator $L$ (cf. (2.2)), which will be shown by solving boundary value problems for difference equations.

2. **Notations and results.** Let us consider the nonlinear analytic Cauchy-Goursat problem

$$\varepsilon D^\beta u = a(x, D^\alpha u), \qquad u = O(x^\beta) \tag{2.1}$$

in a neighborhood of the origin of $C^d$, where $|\alpha| \leqq |\beta|$, $\alpha \neq \beta$ and $\varepsilon$ is a complex constant. We assume that in the expression $a(x, D^\alpha u)$ the function $a(x, y_\alpha)$ satisfies the compatibility condition $a(0,0)=0$ if $\varepsilon = 0$.

We assume

(A.I) If $d \geqq 3$, there exist integers $l_1$ and $l_2$ $(1 \leqq l_1 < l_2 \leqq d)$ such that for every $\alpha$ in (2.1) with $|\alpha| = |\beta|$, either that $\alpha_\nu = \beta_\nu$ for $\nu \neq l_1, l_2$ $1 \leqq \nu \leqq d$ or $(\partial a/\partial y_\alpha)(0,0) = 0$ holds. Here $\alpha = (\alpha_1, \cdots, \alpha_d)$, $\beta = (\beta_1, \cdots, \beta_d)$. Without loss of generality we may assume $l_1 = 1$, $l_2 = 2$. We define a linear operator $L$ associated to (2.1) by

$$L = \varepsilon D^\beta - \sum_{|\alpha| = |\beta|} (\partial a/\partial y_\alpha)(0,0) D^\alpha. \tag{2.2}$$

Then, setting $\lambda = \xi_1/\xi_2$ the characteristic equation for $L$ is given by

$$\varepsilon - \sum_{p=-m}^{n} a_p \lambda^p = 0 \quad (a_p = (\partial a/\partial y_\alpha)(0,0) \qquad \text{if } \alpha - \beta = (p, -p, 0, \cdots, 0)) \tag{2.3}$$

for some integers $m, n \geqq 0$. Since the case $a_p = 0$ ($\forall p > 0$ or $\forall p < 0$) is solved (cf. [2]) we are interested in the case where $a_n a_{-m} \neq 0$ for some $m \geqq 1$ and $n \geqq 1$.