50. *The Ring and Module Forms of the Brauer Correspondence*

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The Brauer correspondence has a ring theoretic definition by means of central characters and the Brauer map (see [3], III. 9). There is a module theoretic definition in terms of restriction of modules (see [2, 5, 1]). The natural question arises as to how the definitions differ. In [1] Alperin showed by non-elementary techniques that the two definitions coincide in the case of primary interest, i.e., for blocks of a subgroup H of G having defect group D with $C_G(D) \leq H$. In [5] Okuyama used elementary techniques to obtain a broadening of Alperin's result. He showed the two correspondences are defined and agree for a block b of a subgroup H of G, if b has multiplicity one as a direct summand of the group algebra FG.

The first section refines the techniques of the Okuyama article. We obtain a strikingly clear contrast between the two definitions in Remark 1.6. One immediate consequence is that whenever both forms of the Brauer correspondence are defined, they coincide (see Theorem 1.7). The remaining two sections address the natural question as to possible differences between the domain of definition of the two forms of the Brauer correspondence. We show that there are differences suggesting the potential usefulness of each definition in a general setting.

1. A general comparison. Fix a finite group G and a subgroup H. Let F be a field of characteristic p. By "block of FH" we mean one of the ideals in the direct sum decomposition of FH into indecomposable two sided ideals; or analogously, an indecomposable $F(H\times H)$-submodule of FH that is a direct summand (see [2] for a complete treatment of the module view). We view FH as a subset of FG.

Definition 1.1. For any block b of FH, define

$$\theta_b : \mathrm{End}_{F(G\times G)}(FG) \rightarrow \mathrm{End}_{F(H\times H)}(b)$$

by $\theta_b(f)=\pi_b f \rho_b$ where $\pi_b : FG \rightarrow b$ is the $F(H\times H)$-module projection and $\rho_b : b \rightarrow FG$ is the $F(H\times H)$-module injection. Also let

$$\bar{\theta}_b : \mathrm{End}_{F(G\times G)}(FG) \rightarrow \mathrm{End}_{F(H\times H)}(b)/J(\mathrm{End}_{F(H\times H)}(b))$$

be θ_b composed with the canonical map.

For b a block of FH, b^G indicates its ring theoretic Brauer correspondent. In general $\bar{\theta}_b$ is only a F-vector space homomorphism.