47. *A Canonical Form of a System of Microdifferential Equations with Non-Involutory Characteristics and Branching of Singularities*

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We study a system $\mathcal{M}$ of microdifferential (=pseudodifferential) equations. We assume that the characteristic variety V of $\mathcal{M}$ is the union of two regular submanifolds with non-involutory intersection. We also assume that $\mathcal{M}$ has regular singularities along V. (Precise assumptions will be given below.) In § 1, we give a canonical form of $\mathcal{M}$ in the complex domain. Applying this result, we study in § 2 the branching of supports of microfunction solutions of $\mathcal{M}$ under the additional assumption that $\mathcal{M}$ is hyperbolic. Details of this article will appear elsewhere.

§ 1. A canonical form of a system with regular singularities along its non-involutory characteristics. Let X be an n-dimensional complex manifold and T^*X be its cotangent bundle. We identify the zero section of T^*X with X. Let $z=(z_1, \cdots, z_n)$ be a local coordinate system of X. Then $(z, \langle\zeta, dz\rangle)=(z, \zeta)=(z_1, \cdots, z_n, \zeta_1, \cdots, \zeta_n)$ denotes a point of T^*X. We denote by $\mathcal{E}_X$ the sheaf on T^*X of microdifferential operators (of finite order). Note that $\mathcal{E}_X$ is denoted by $\mathcal{P}_X^f$ in [6]. Let $\mathcal{O}(j)$ be the sheaf on T^*X of holomorphic functions homogeneous of degree j with respect to the fiber coordinates. We denote by $\mathcal{E}(j)$ the sheaf of microdifferential operators of order at most j. There is a natural homomorphism

$$\sigma_j : \mathcal{E}(j)\to\mathcal{O}(j)\cong\mathcal{E}(j)/\mathcal{E}(j-1).$$

If $P\in\mathcal{E}(j)-\mathcal{E}(j-1)$, we call $\sigma(P)=\sigma_j(P)$ the principal symbol of P. For a homogeneous (=conic) involutory analytic subset V of T^*X-X, we set $I_V(j)=\{f\in\mathcal{O}(j)\,;f|_V=0\}$. Then $\mathcal{O}_V(0)=\mathcal{O}(0)/I_V(0)$ is a coherent sheaf of rings on V. We set $\mathcal{J}_V=\{P\in\mathcal{E}(1)\,;\sigma_1(P)\in I_V(1)\}$ and denote by $\mathcal{E}_V$ the subring of $\mathcal{E}_X$ generated by $\mathcal{J}_V$.

Let $\omega=\zeta_1dz_1+\cdots+\zeta_ndz_n$ be the fundamental 1-form on T^*X. A homogeneous involutory submanifold of T^*X-X is said to be regular if the pull back of ω to it vanishes nowhere.

For a submanifold W of T^*X and a point p of W, we say that p is a point of rank $2r$ in W if the rank of the skew-symmetric bilinear form $d\omega$ on T_pW is of rank $2r$. If each point of W is a point of rank $2r$ in W, we say that W is of rank $2r$ and write rank $W=2r$.