45. *Hopf Bifurcation of Semilinear Evolution Equations*

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(Communicated by Kôsaku YOSIDA, M. J. A., April 13, 1981)

1. Introduction and the assumptions. The present paper is concerned with the two problems. The first problem is the Hopf bifurcation problem for a semilinear evolution equation in a real Banach space X (with norm $\| \ \|$) with a real parameter λ;

(E) $$du/dt = Lu + N(u, \lambda) \qquad t>0.$$

The second one is to determine a local ω-limit set of a solution $u(t, x_0)$ of a semilinear evolution equation in X;

(E′) $$du/dt = Lu + N(u) \qquad t>0$$

with an initial value: $u(0)=x_0$. Here we assume

Assumption 1. L is the generator of the holomorphic semigroup, having $\pm i$ as isolated eigenvalues with the algebraic multiplicity one and the other spectrum $\sigma'(L)$ of L being properly contained in the left half-(complex)plane;

$$\sup_{\mu \in \sigma'(L)} \operatorname{Re} \mu < -c$$

(c: a positive constant).

Assumption 2. $N(x, \lambda)$ is a C^3-mapping of a neighborhood of 0 in $X \times R^1$ into X such that $N(0, 0)=0$, $D_x N(0, 0)=0$. ($D_x N(0, 0)$ means the Fréchet derivative of $N(x, \lambda)$ with respect to x at $x=0$, $\lambda=0$.)

Assumption 2′. $N(x)$ is a C^3-mapping of a neighborhood V of 0 (in X) into X such that $N(0)=0$.

Before stating our results, we shall give the definition of a local ω-limit set of a solution $u(t, x_0)$ of (E'). Let U_1, U_2 be neighborhoods of 0 with $U_1 \subset U_2 \subset V$. For $x_0 \in U_1$ we define a local ω-limit set $\Omega_{U_1,U_2}(x_0)$ of a solution $u(t, x_0)$ of (E') by

$$\Omega_{U_1,U_2}(x_0) = \begin{cases} \bigcap_{s \geq 0} \text{closure } \{u(t, x_0)\,;\, t \geq s\} & (\text{if } u(t, x_0) \in U_2,\ t \geq 0) \\ \phi & (\text{otherwise}) \end{cases}.$$

2. Results. Theorem 1. *Under Assumptions* 1 *and* 2, *if a null solution* 0 *of* (E) *changes its stability at* $\lambda=0$, *then non-stationary periodic orbits bifurcate from* $(x, \lambda)=(0, 0)$.

Theorem 2. *Under Assumptions* 1 *and* 2′, *there exists a neighborhood* $U_1(\subset V)$ *of* 0 *such that if* $\sup_{x \in V} \|D_x N(x)\|$ *is sufficiently small, then for some* $U_2(U_1 \subset U_2 \subset V)$ *and for any* $x_0 \in U_1$ *with* $\Omega_{U_1,U_2}(x_0) \neq \phi$, $\Omega_{U_1,U_2}(x_0)$ *consists only of a periodic orbit* $\gamma(x_0)$ *of* (E′) *in* U_2 ($\gamma(x_0)$ *may be* $\{0\}$).