39. *Moduli of Anti-Self-Dual Connections on Kähler Manifolds*

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0. The aim of this note is to give a brief proof of the following theorem which concerns the moduli space of anti-self-dual connections on a Kähler manifold.

Theorem. *Let M be a compact Kähler 2-manifold of positive scalar curvature. Let P be a G-principal bundle over M, and E a complex vector bundle associated to P, where G is a compact semi-simple Lie group. If a G-connection on E is anti-self-dual and irreducible, then the space of its infinitesimal deformations is of dimension*

$$-Pont_1(\mathfrak{g}_P^C)-\frac{1}{2}\dim G(\chi+\tau),$$

where $Pont_1(\mathfrak{g}_P^C)$ is the first Pontrjagin class of the adjoint bundle $\mathfrak{g}_P^C$, χ and τ are the Eular number and the signature of M respectively.

In the case when M is a self-dual Riemannian 4-manifold, the following remarkable theorem is obtained by Atiyah, Hitchin and Singer.

Theorem (Atiyah, Hitchin and Singer [1]). *Let M be a compact oriented self-dual Riemannian 4-manifold of positive scalar curvature. Let P be a G-principal bundle, where G is a compact semi-simple Lie group. Then, the space of moduli of irreducible self-dual connections on P is either empty or a manifold of dimension*

$$Pont_1(\mathfrak{g}_P^C)-\frac{1}{2}\dim G(\chi-\tau).$$

Here an oriented Riemannian 4-manifold is called self-dual if its Weyl's conformal curvature tensor is self-dual.

The proof of this theorem was proceeded with three parts as follows; (1) to compute the dimension of the space of infinitesimal deformations by using the Atiyah-Singer index theorem together with a vanishing theorem, (2) to use the method of Kuranishi in order to obtain a local homeomorphism from the space to a local moduli space and (3) to show that these local spaces give local charts on the global moduli space.

While the dimension of the space of infinitesimal deformations in our theorem is calculated in a similar manner to the method used in